LHC Physics: QCD tools

P. Nason
INFN, Sez. of Milano Bicocca

Outline

Introduction:

- The problem: how to see quarks from hadrons
- QCD calculations: always an all orders problem
- Quarks as jets

Fixed order calculations:

- Tree level matrix elements: automatic generators
- How to define a jet cross section using a tree level ME

Shower Monte Carlo

- Basics
- Merging Multi-parton matrix elements and showers
- CKKW approach

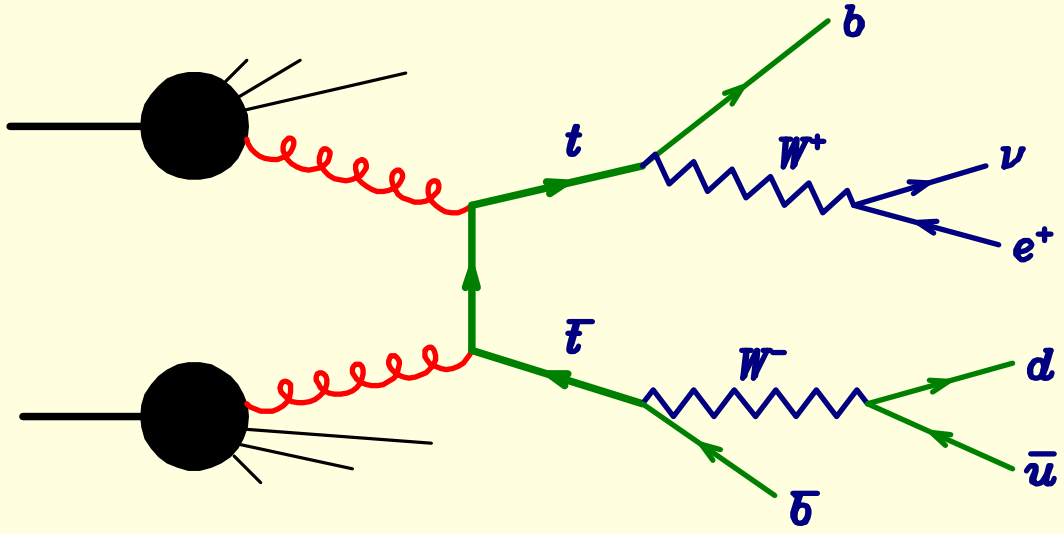
Calculations at Next-to-Leading orders

- Calculation of complex processes: status
- Problems with NLO calculations
- Merging NLO and showers
- `MC@NLO` and `POWHEG`.

Quarks from hadrons

Discovery physics focussed upon stable elementary objects, such as photons, leptons, gluons and quarks.

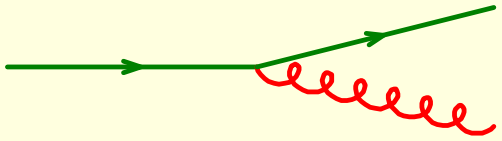
Ideally, we would need leptons and quarks colliders, and experiments that detect photons, leptons, gluons and quarks. For photons and leptons, this is almost the case. For gluons and quarks quarks, things are much more difficult;



$t\bar{t}$ production: incoming protons can be seen as broad-band beams of coloured particles. Final state quarks manifest themselves as hadron jets.

QCD calculations: all orders always needed

Even in the most elementary QCD process we always have contributions of arbitrarily high order coming in. Low angle splitting processe of final state quarks and gluons yield contributions of order 1



Splitting processes contributes as

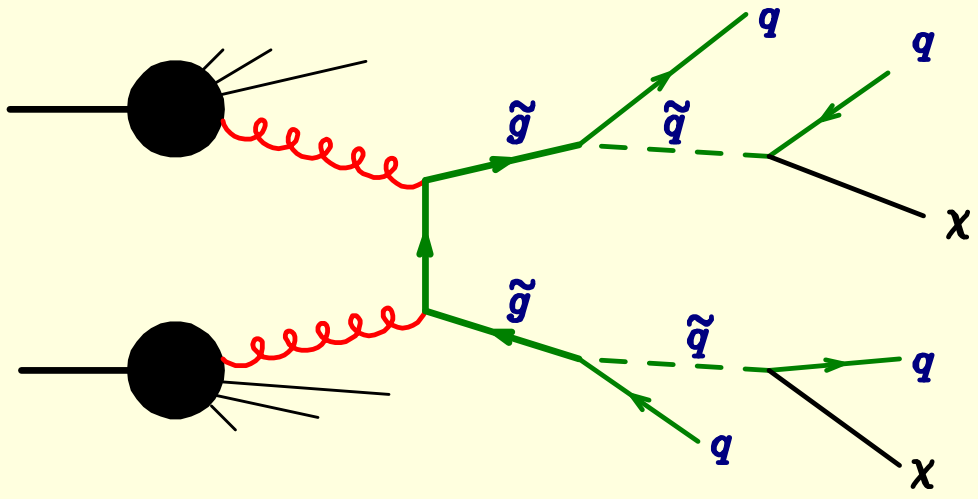
$$\approx \int \alpha_s(t) \frac{dt}{t} \approx \alpha_s \log Q^2 \approx 1$$

An elementary diagram containing final state quarks and gluons should be interpreted as an inclusive process, where final state quarks and gluons undergo an arbitrary number of splitting processes.

So: when talking about a tree level process in QCD, always keep in mind that it is an inclusive process. The divergences in the sum of real splitting processes and virtual corrections cancel because of the Kinoshita-Lee-Nauenberg mechanism. Quarks evolve into multiparticle systems (jets) in the final state.

Tree level processes for LHC

Even the "simple" graph for $t\bar{t}$ production at hadron colliders is not so easy to compute, if all angular correlations of decay products are taken into account.
Background to (semileptonic) $t\bar{t}$ production: $W + 4\,j$, very difficult!
Discovery example: gluino production



If $\chi = \chi^0$: MET+4 jets;
if $\chi = \chi^\pm \to W^\pm + \chi^0$:
MET+ up to 8 jets;

$W + \text{jets}$,
$Z + \text{jets}$ $(Z \to \nu\bar{\nu})$,
$t\bar{t} + \text{jets}$
are all backgrounds;

Can we compute such complicated processes, even at the tree level?

Calculation of Complex Processes: LO (tree Level) Matrix Elements

Many available programs can do automatic evaluation of LO cross sections.

1. Helicity amplitudes (HELAS, Hagiwara, Kanzaki, Murayama, Watanabe; MadGraph, Maltoni, Stelzer)
2. Behrends-Giele recursion relations (VecBos)
3. other recursive methods, (ALPHA, Caravaglios, M.Moretti)
   - ALPGEN, Mangano, Moretti, Piccinini, Pittau, Polosa
   - HELAC, Kanaki, Papadopoulos
4. CSW recursion (from twistors), Cachazo, Svrček, Witten,2004, Dixon, Glover, Khoze, Badger, Bern, Forde, Kosower, Mastrolia
5. BCFW recursion, Britto, Cachazo, Feng, Witten,2004
   +masses: Badger, Glover, Khoze, Svrček; Schwinn, Weinzierl

Comparison of algorithms

CSW and BCF yield more compact expressions.

Comparison of automated algorithms by

Duhr, Hoche, F.Maltoni, Jun.06; also Dinsdale, Ternick, Weinzierl, Feb.06;

BG=Berends-Giele, CSW=Cachazo-Svrček-Witten, BCF=Britto-Cachazo-Feng

CO=Colour ordered, CD=Colour dressed (i.e. full amplitude)

Final state	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
$2g$	0.24	0.28	0.28	0.33	0.31	0.26
$3g$	0.45	0.48	0.42	0.51	0.57	0.55
$4g$	1.20	1.04	0.84	1.32	1.63	1.75
$5g$	3.78	2.69	2.59	7.26	5.95	5.96
$6g$	14.20	7.19	11.9	59.10	27.80	30.60
$7g$	58.50	23.70	73.6	646.00	146.00	195.00
$8g$	276.00	82.10	597	8690.00	919.00	1890.00
$9g$	1450.00	270.00	5900	127000.00	6310.00	29700.00
$10g$	7960.00	864.00	64000		48900.00	

Berends-Giele (comparable to ALPGEN, HELAC) still faster ...

summarizing (LO):

- General purpose ME generators for SM and MSSM tree level processes are available (example: Madgraph, any process, not very fast)
- Very fast generators, capable to add several gluons in the final state already available. Example: ALPGEN, processes added by authors

$WQ\bar{Q}$ + up to 4 jets	$Q\bar{Q}\,H$ + up to 4 jets
$Z/\gamma + Q\bar{Q}$ + up to 4 jets	Inclusive N jets, with N up to 6
W + up to 6 jets	$N\gamma$ + M jets
$W + c$ + up to 5 jets	Single top
Z + up to 6 jets	W + photons + jets
$nW + m\,Z + k\,H + l\gamma$ + up to 3 jets	$WQ\bar{Q}$ + photons + jet
$Q\bar{Q}$ + up to 6 jets	$Q\bar{Q}$ + M-photons + N-jets
$Q\bar{Q} + Q'\bar{Q}'$ + up to 4 jets	Higgs + up to 5 jets

Total automation of fast techniques desirable (not far)

Play with MadGraph

The "not so fast" methods are still much faster than computing feynman graphs in the traditional way.
They are based upon a very simple idea: compute amplitudes (not squared amplitudes), with a purely numerical implementation the Feynman rules.
Thus: for a final state fermion, build up a helicity complex spinor.
For a final state vector, build the complex polarization vector $\epsilon^{\mu}_{\pm,0}$.

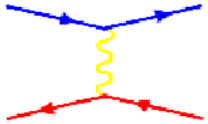
To compute a tree graph, we only need to be able to compute the merging of these objects: two four component spinors merging into a four component vector, a spinor and a vector merging into a spinor, and so on.

This method allows to compute cross sections like $W+$ up to 4 partons
(compare it to Alpgen: W + up to 6 partons!!).

Visit the MadGraph web site: http://madgraph.hep.uiuc.edu/
and get the code for simple processes to see how it works!

MadGraph Version 4

UCL UIUC Fermi
by the MG/ME Development team



Generate Process | Register | Tools | My Database | Cluster Status | Downloads (needs registration) | Wiki/Docs | Admin

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Generate Code On-Line

To improve our web services we now request that you register. Registration is quick and free. You may register for a password by clicking here

Code can be generated either by:

I. Fill the form:

Model: SM — Model descriptions

Input Process: — Examples

Max QCD Order: 99

Max QED Order: 99

p and j definitions: p=j=d u s c d~ u~ s~ c~ g

sum over leptons: l+ = e+, mu+ ; l- = e-, mu- ; vl = ve, vm ; vl~ = ve~, vm~

Submit

Problems with tree level calculations

Look for example at $W + j$. A matrix element process that gives rise to it is $u\,\bar{d} \to W^+ + g$. But this process should be seen as an inclusive one (the final state gluon can undergo several splitting processes at small angle).

Consider now $u\,\bar{d} \to W^+ + 2g$. How is it distinguished from $W + g$?

We should ask that the two gluons are sufficiently far apart that they do not constitute a single jet. For example, we may require that the invariant mass M_{12} (or an equivalent measure) of the two-gluon system is above a given cut.

Also an initial state quark may have originated from an initial quark splitting into a quark and gluon. This yields corrections of order 1 when the gluon is nearly parallel to the incoming line. Thus, we should also ask that the gluon has transverse momentum above a given cut.

Matrix element samples of events with 1, 2, 3 or more partons in the final state are build by requiring a $p_T > M$, and $M_{ij} > M$.

Remember: $W + n\,j$ cross section in this sample also represents events where more than n partons were produced, either with $p_T < M$ or with $M_{ij} < M$ for some parton pairs.

M should be large enough for perturbation theory to hold, i.e. $\alpha_s(M)$ should be small. But this is not enough. corrections go like $\log\frac{Q}{M}\,\alpha_s(M)$!

Look for example at $e^+e^- \to q\,\bar{q}$. We know that

$$\sigma(e^+e^- \to \text{Hadrons}) = \sigma_0(e^+e^- \to q\,\bar{q}) + \mathcal{O}(\alpha_s(Q)).$$

This is because radiation of final state lines, giving corrections of order

$$\alpha_s \int_\Lambda^Q \frac{dt}{t} \approx \alpha_s \log\frac{Q}{\Lambda} \approx 1$$

cancel agains corresponding virtual corrections V

$$V + \alpha_s \int_\Lambda^Q \frac{dt}{t} \approx \alpha_s \ll 1, \quad \text{so} \quad V \approx -\,\alpha_s \int_\Lambda^Q \frac{dt}{t}$$

But if we limit the real radiation to $t < M$, we get

$$V + \alpha_s \int_\Lambda^M \frac{dt}{t} = -\,\alpha_s \int_M^Q \frac{dt}{t} \approx -\,\alpha_s \log\frac{Q}{M}$$

(A more precise analysis would show that in fact it is $\log^2$ rather than $\log$)

So: $\log\frac{Q}{M}\,\alpha_s(M)$ should be small for the ME approach to work, and M cannot be to far below the scale Q of the process in question.

Clearly unsatisfactory: we can have $\log\frac{Q}{M}\,\alpha_s(M) \approx 1$ even with $M \gg \Lambda$!

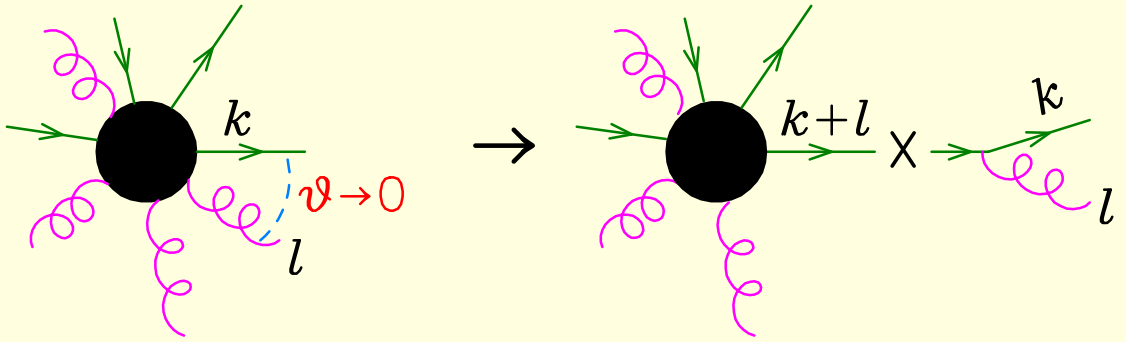
Is it possible to give a more exclusive description, bringing M closer to the GeV limit?

The answer is yes, in the framework of shower algorithms.

Shower basics: Collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 d\Phi_n \, \frac{\alpha_s}{2\pi} \, \frac{dt}{t} \, P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

t : hardness (either virtuality or p_T^2 or $E^2\theta^2$ etc.)

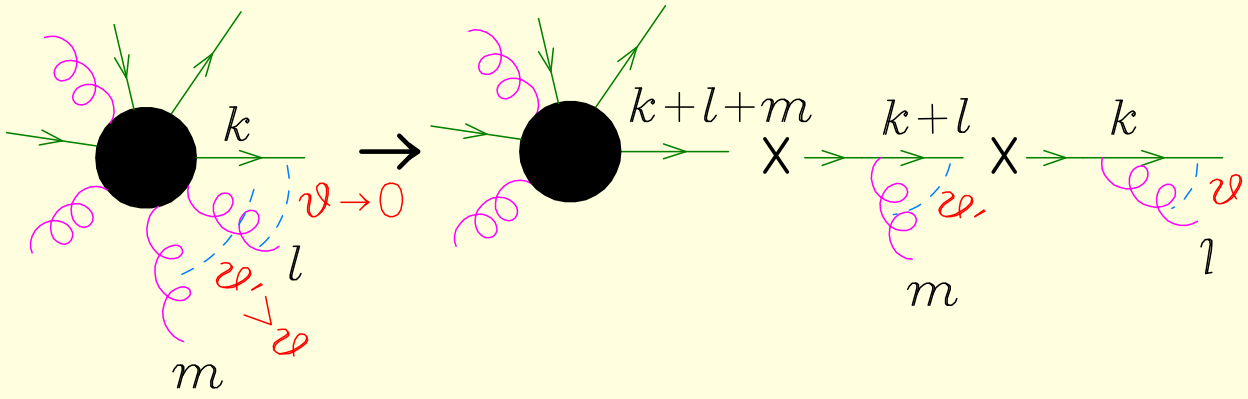
$z = k^0/(k^0+l^0)$: energy (or $p_\parallel$, or p^+) fraction of quark

$P_{q,qg}(z) = C_F \dfrac{1+z^2}{1-z}$: Altarelli – Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$
with $\theta' > \theta$



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P_{q,qg}(z')\,dz' \frac{d\phi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P_{q,qg}(z)\,dz \frac{d\phi}{2\pi}\theta(t'-t)$$

Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

$$\left(\frac{\alpha_s}{2\pi}\right)^m \int_{\theta_{\min}} \frac{d\theta_1}{\theta_1} \int_{\theta_1} \frac{d\theta_2}{\theta_2} \cdots \int_{\theta_{m-1}} \frac{d\theta_m}{\theta_m} \;\propto\; \frac{\log^m \frac{1}{\theta^2_{\min}}}{m!} \approx \left(\frac{\alpha_s}{2\pi}\right)^m \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}$$

where we have taken $\theta_{\min} \approx \Lambda/Q$; (Leading Logs) This is of order 1!

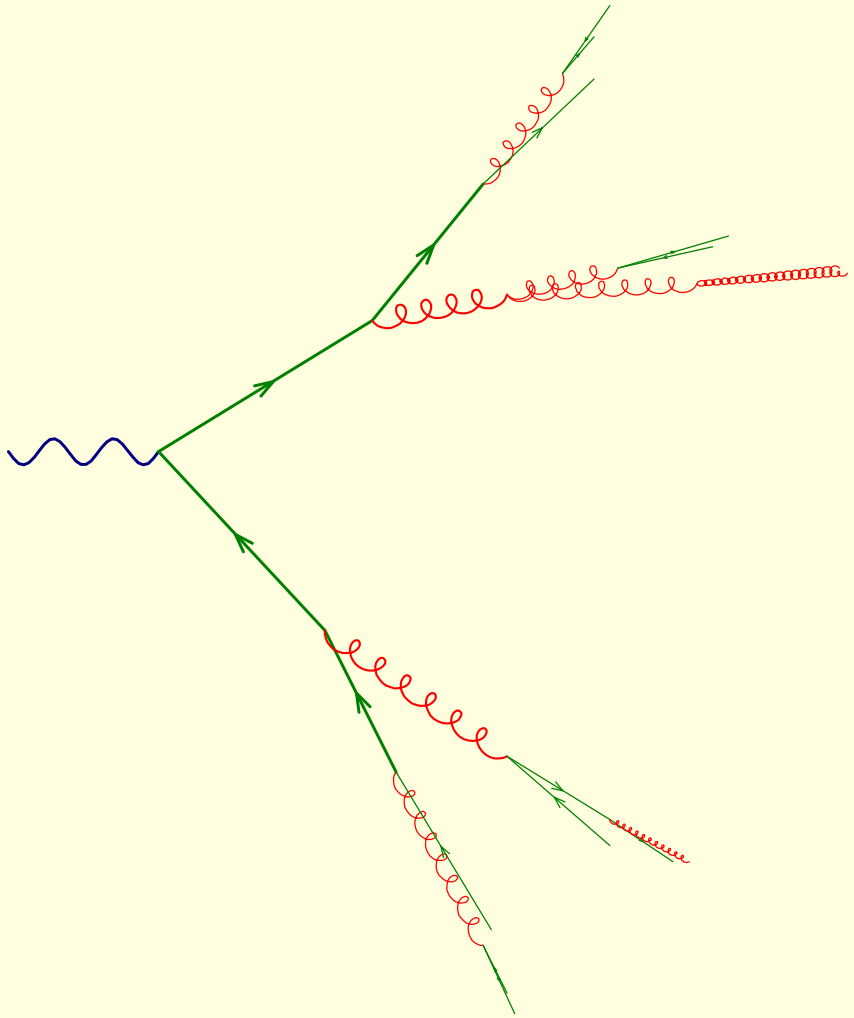
Typical dominant configuration at very high Q^2

Besides $q \to qg$, also $g \to gg$, $g \to q\bar{q}$ come into play.

Typical configurations: intermediate angles of order of geometric average of upstream and downstream angles.

Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.

As relative momenta become smaller α_s becomes bigger, and this picture breaks down.



For a consistent description:

include virtual corrections to same LL approximation

One can show that the effect of virtual corrections is given by

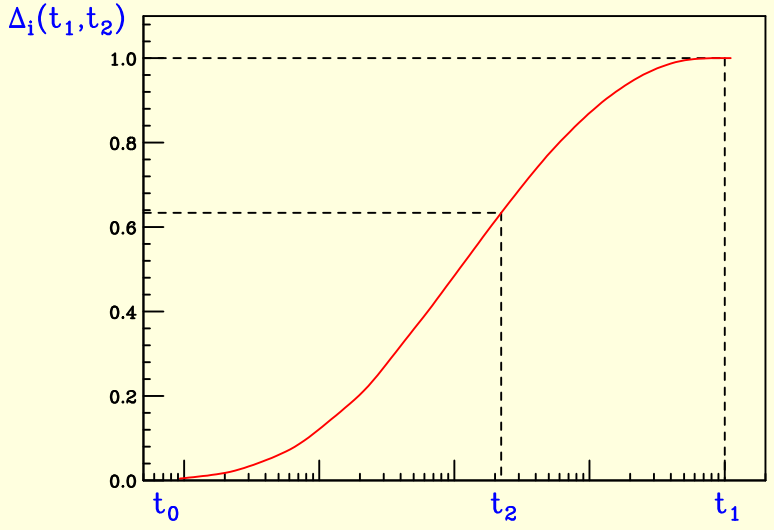
- Let $\alpha(\mu) \Longrightarrow \alpha(t)$ in each vertex, where t is the hardness of the vertex (i.e. hardness of the incoming line)
- For each intermediate line include the factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} \, P_{i,jk}(z) \right]$$

  where t_h is the hardness of the vertex originating the line, and t_l is the hardness of the vertex where the line ends.

Sudakov form factor

$$\Delta_i(t_h, t_l) = \exp\left[-\sum_{(jk)} \int_{t_l}^{t_h} \frac{dt'}{t'} \int dz \, \frac{\alpha_s(t')}{2\pi} P_{i,jk}(z) \right]$$



As t_l becomes small the exponent tend to diverge, and $\Delta_i(t_h, t_l)$ approaches 0. In fact, because of $\alpha_s(t)$, we must stop at $t_0 \gtrsim \Lambda_{\mathrm{QCD}}$.

Final Recipe

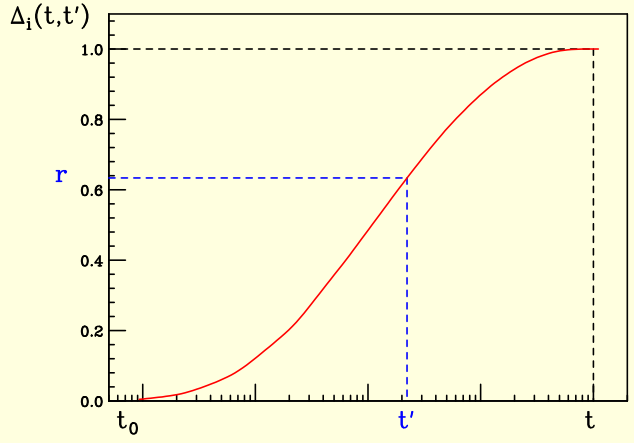
- Consider all tree graphs.
- Assign ordered hardness parameters t to each vertex.
- Include a factor

$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}$$

  at each vertex $i \to jk$.
- Include a factor $\Delta_i(t_1, t_2)$ to each internal line with a parton i, from hardness t_1 to hardness t_2.
- Include a factor $\Delta_i(t, t_0)$ on final lines (t_0: IR cutoff)

Most important: the shower recipe can be easily implemented as a computer code!
Shower Algorithm:

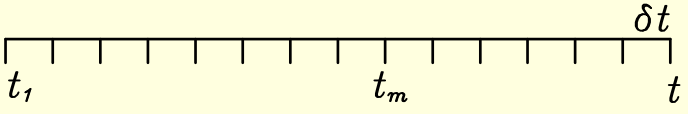
- Generate a uniform random number $0 < r < 1$;
- Solve the equation $\Delta_i(t, t') = r$ for t';
- If $t' < t_0$ stop here (final state line);
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly;
- restart from each branch, with hardness parameter t'.



Probabilistic intepretation: branching probability of line of flavor i

$$dP(t_1,t) = \underbrace{\exp\left[-\sum_{(jk)}\int_t^{t_1}\frac{dt'}{t'}\int dz\,\frac{\alpha_s(t')}{2\pi}P_{i,jk}(z)\right]}_{\Delta(t_1,t)}\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}$$

break up t_1, t into small subintervals:



$$dP(t_1,t) = \left[\prod_m \left(\underbrace{1-\sum_{(jk)}\frac{\delta t}{t_m}\int dz\,\frac{\alpha_s(t_m)}{2\pi}P_{i,jk}(z)}_{\text{No emission prob. in } t_m, t_m+\delta t}\right)\right]\underbrace{\frac{\alpha_s(t)}{2\pi}P_{i,jk}(z)\,\frac{\delta t}{t}\,dz\,\frac{d\phi}{2\pi}}_{\text{emission prob. in } t, t+\delta t}$$

So: the probability for the first branching at hardness t is the product of the non-emission probability $\Delta(t_1,\ t)$ in all hardness intervals between t_1 and t, times the emission probability at hardness t.

(more or less) obvious consequences:

- The total branching probability plus the no-branching probability is 1; mathematically

$$\int_{t_0}^{t_1} dP(t_1, t') = \int_{t_0}^{t_1} d\Delta_i(t_1, t') = 1 - \Delta_i(t_1, t_0)$$

- The Sudakov form factor $\Delta_i(t_1, t)$ is the no-branching probability from scale t_1 down to the scale t.

- The branching probability is independent of what happens next (because the total probability of what happens next is 1).

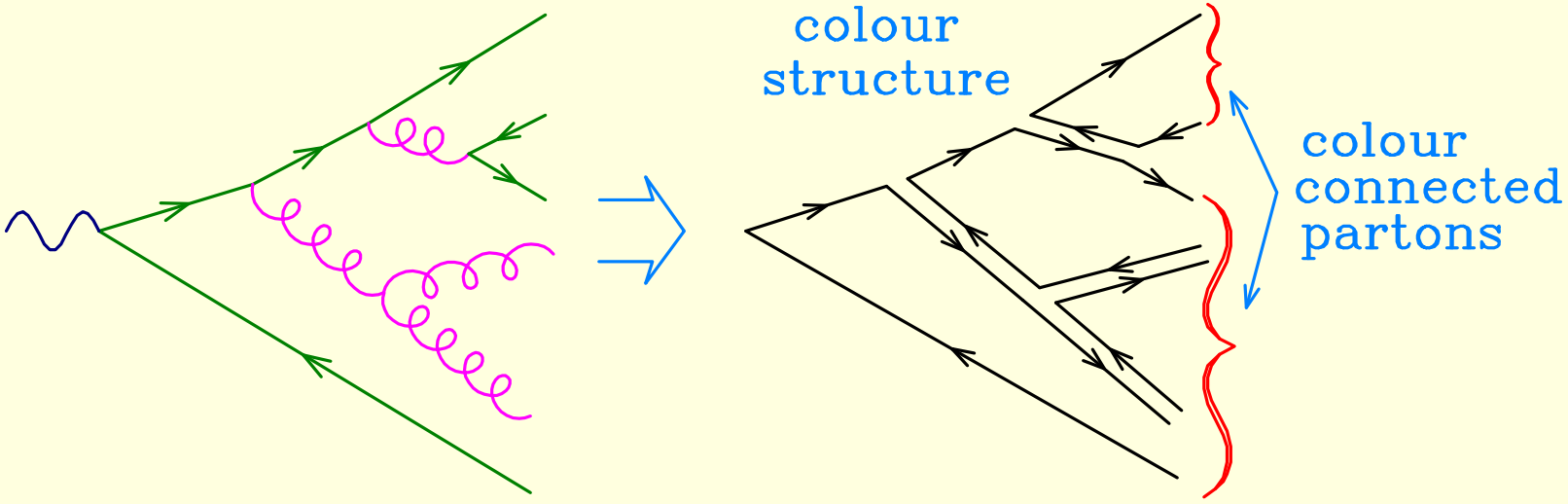
This property is often called unitarity of the shower. It is a consequence of the Kinoshita-Lee-Nauenberg theorem: collinear divergence must cancel in the inclusive cross section.

COLOUR AND HADRONIZATION

SMC's assign colour labels to partons.

Only colour connections are recorded (as in large N limit).

Initial colour assigned according to hard cross section.



Colour assignements are used in the hadronization model.

Most popular models: Lund String Model, Cluster Model.

In all models, color singlect structures are formed out of colour connected partons, and are decayed into hadrons preserving energy and momentum.

Large angle emission

A disturbing feature of SMC's: the hardest jet generated in the shower is not really collinear in about 10% of the events (i.e. $\mathcal{O}(\alpha_s)$). Thus, the gross feature of the event is wrongly described by the SMC in 10% of the cases.

So: the Shower algorithm describes well small angle radiation, but fail for the small fraction of large angle emission events.

This small fraction is precisely what is relevant for QCD background to new physics processes.

For example, in $\tilde{g}\tilde{g}$ production with $\tilde{g} \to (\tilde{q} \to q\chi)\, q$, the quark system has large invariant mass, and the small p_T region is not priviledged.

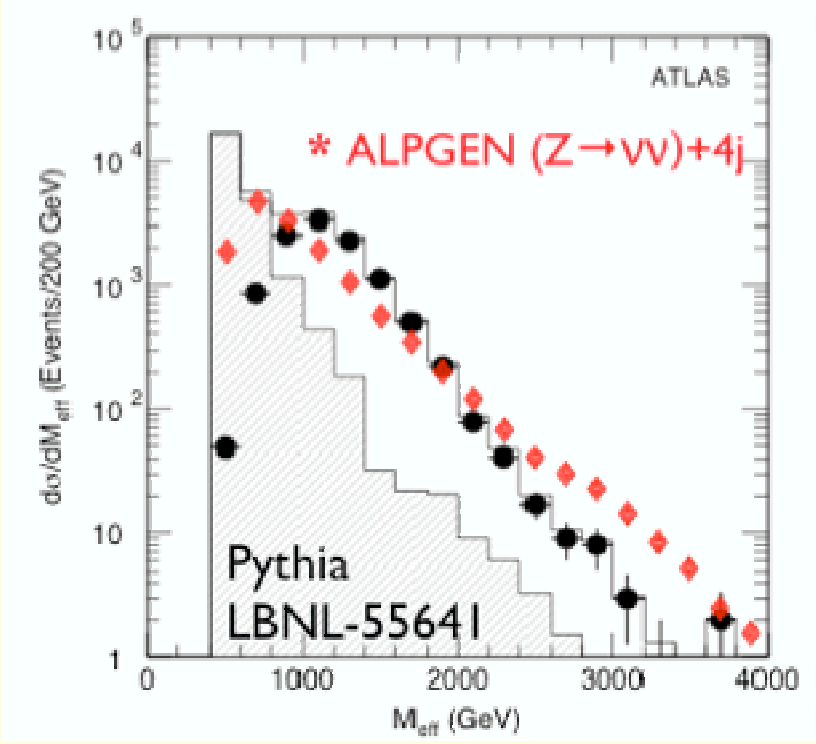
Thus, although showers describe well the bulk of events, they fail for the most important ones from the point of view of backgrounds to new physics.

Multi-parton Matrix Elements

With LHC physics: cannot trust collinear approximation for multi-jet background to complex processes

The use of exact ME is mandatory (Gianotti,Mangano, 05)

$M_{\rm eff}$ distribution for a potential multijet+$E_T^{\rm miss}$ SUSY signal
dark circles: signal
Shaded area: MC background



Can we MERGE matrix element calculations with parton shower algorithms, to get the best of both world?

This has been a long-standing problem; a consistent solution has been formulated by Catani, Krauss, Kuhn and Webber in 2001.

Historical approach: CKKW

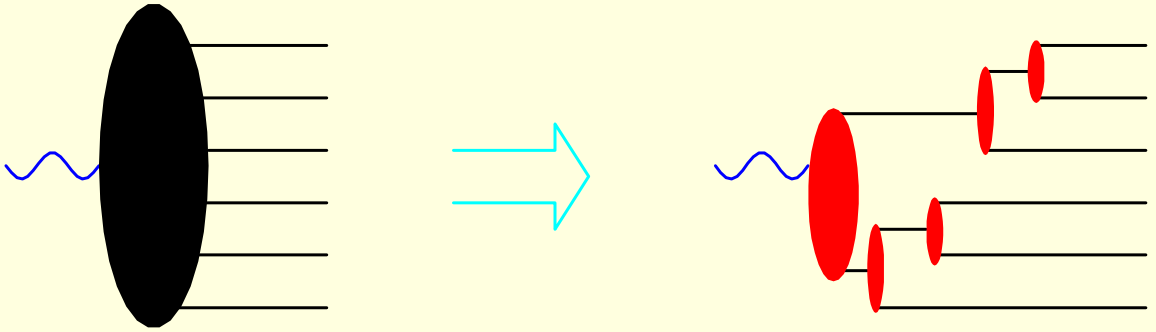
Catani, Krauss, Küen, Webber (2001), (in e^+e^- annihilation).

In a nut-shell:

- Correct ME calculations when they approach the collinear region, so that they reproduce Shower results
- Let the Shower take care of radiation with $M < M_{\rm cut}$

In a better approximation

- Build a sample of ME events, generated with a probability proportional to the corresponding cross section. At this stage, use a fixed reference value of the strong coupling $\alpha_s(M)$. Events are generated with a cut M on the t of parton pairs, and on the p_T of each parton.
- Clusterize ME partons to reconstruct a shower skeleton
  (by pairing up particles that yield smallest t recursively)



Red blobs have decreasing t values

You can think of t as the virtuality of the pair, but other definitions are possible.

- Evaluate ME couplings $\alpha_s(t)$ at scales t of vertices in shower skeleton
- Assign Sudakov form factors $\Delta(t, t')$ to the skeleton intermediate lines (as in Shower MC)
- Reject the event with a probability $\prod \frac{\alpha_s(t)}{\alpha_s(M)} \prod \Delta(t, t')$
- Pass the event to a shower Monte Carlo, with the instruction to shower each final state line, with shower initial condition equal to M.

Events generated in this way reduce to what a shower MC would do for small angles. Furthermore, the procedure should have only small M dependence. By moving M, the amount of job performed by the ME and by the shower changes, but this should not make much difference if M is small enough.

This is not yet the full content of the CKKW algorithm. The really difficult part has to do with the handling of soft-collinear radiation.

CKKW: details

CKKW relies upon the theory of soft-collinear radiation in QCD, through the following steps:

A) Theory of multiple emissions in the soft collinear regions (Mueller, 1981; Ermolaev and Fadin, 1981; Bassetto, Ciafaloni, Marchesini, etc.)

B) k_T-cluster multiplicity calculable at the NLL level in framework A) (Catani, Dokshitzer, Olsson, Turnock and Webber, 1991)

C) k_T-cluster cross section is improved with Sudakov form factors and running α_s (i.e. dominant virtual corrections) from step B)

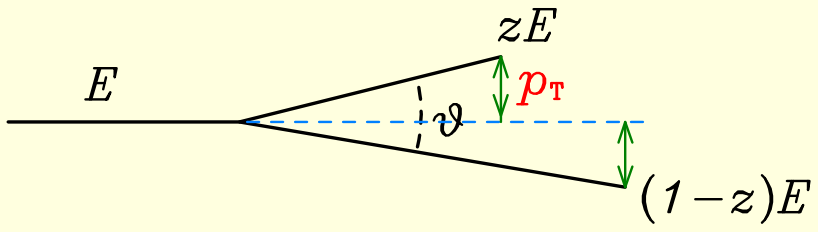
D) Completion of the algorithm with subsequent angular ordered shower in collinear and soft approximation

Soft divergences and double log region

$z \to 1$ ($z \to 0$) region problematic: for $z \to 1$: $P_{qq}, P_{gg} \propto \frac{1}{1-z}$

Choice of hardness variable makes a difference

$$\begin{aligned}
\text{virtuality:} \quad t &\equiv E^2 z(1-z) \overbrace{\theta^2/2}^{1-\cos\theta} \\
p_T^2\text{:} \quad t &\equiv E^2 z^2 (1-z)^2\, \theta^2 \\
\text{angle:} \quad t &\equiv E^2\, \theta^2
\end{aligned}$$



Notice, from the figure, for small p_T: $\theta \approx \frac{p_T}{zE} + \frac{p_T}{(1-z)E} = \frac{p_T}{z(1-z)E}$

$$\underbrace{\int \frac{dt}{t} \int_0^{1-\sqrt{t}/E} \frac{dz}{1-z}}_{\text{virtuality: } 1-z > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{4}; \quad \underbrace{\int \frac{dt}{t} \int_0^{1-t/E^2} \frac{dz}{1-z}}_{p_T^2\text{: } (1-z)^2 > t/E^2} \approx \frac{\log^2 \frac{t}{E^2}}{2}; \quad \underbrace{\int \frac{dt}{t} \int_0^{1} \frac{dz}{1-z}}_{\text{angle}} \approx \log t \log \Lambda$$

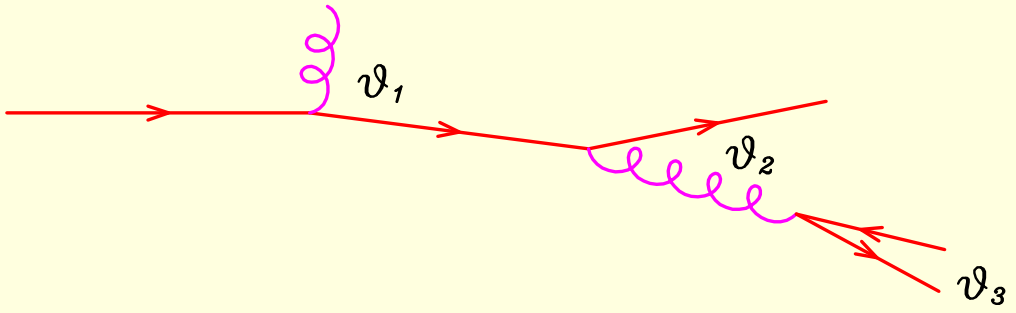
Sizeable difference in double log structure!

At double log level:
angular ordering is the correct choice (Mueller 1981)

$$\frac{d\theta}{\theta}\frac{\alpha_s(p_T^2)}{2\pi}P(z)\mathrm{dz}$$

$$\theta_1 > \theta_2 > \theta_3 \ldots$$

$$p_T^2 = E^2 z^2 (1-z)^2\,\theta^2$$

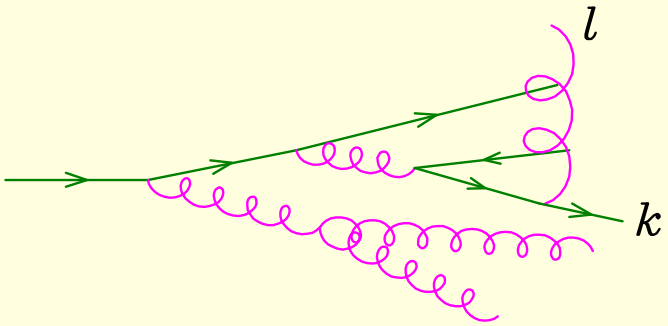


$\alpha_s(p_T)$ for a correct treatment of charge renormalization in soft region.

$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t} \frac{dt}{t} \int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz \frac{\alpha_s(p_T)}{2\pi} \sum_{(jk)} P_{i,jk}(z) \right]$$

$$\approx \exp\left[-\frac{c_i}{4\pi b_0} \left\{ \log\frac{t}{\Lambda^2} \log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0} \right\}_{t'}^{t} \right] \qquad (c_q = C_F, c_g = 2C_A)$$

Sudakov damping stronger than any power of t.

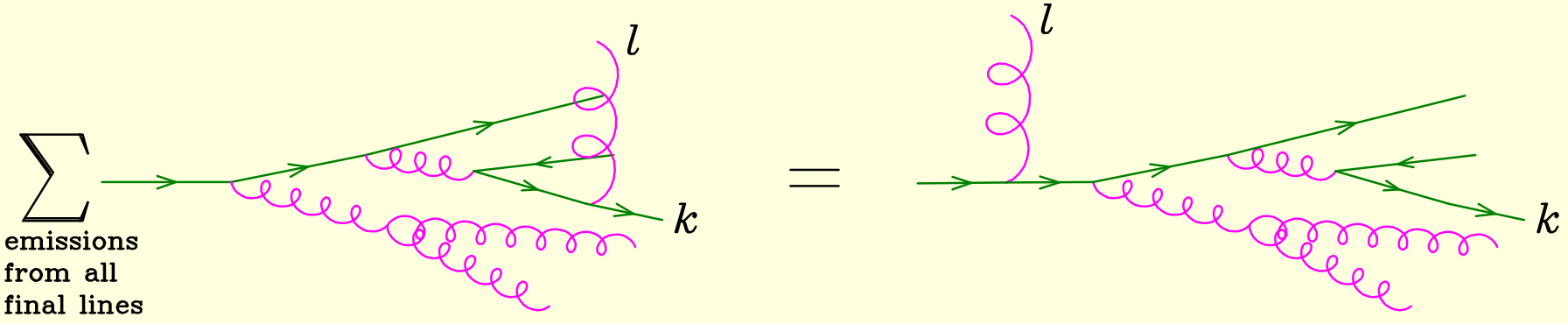


With virtuality ordering:

Soft emissions give small virtuality.

At end of shower, large amount of unrestricted (all angles) soft radiation

But soft gluons emitted at large angles from final state partons add coherently!

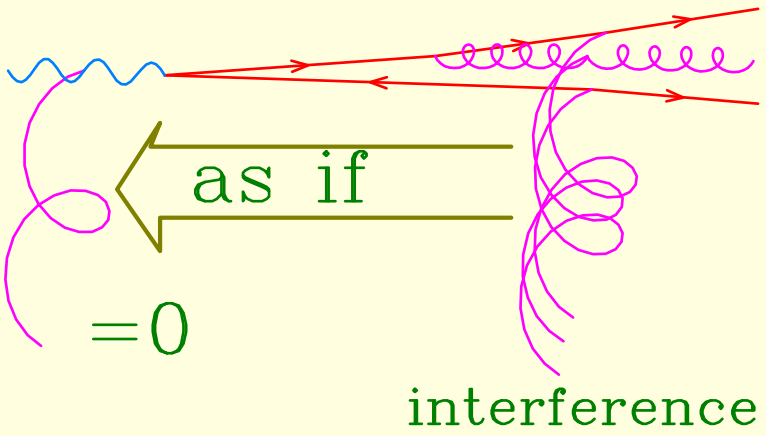


large angle, high energy: already ordered in angle

large angle, small energy: should be reordered by angle;

Thus: order in angle

Look at the example:



Angular ordering accounts

for soft gluon interference.

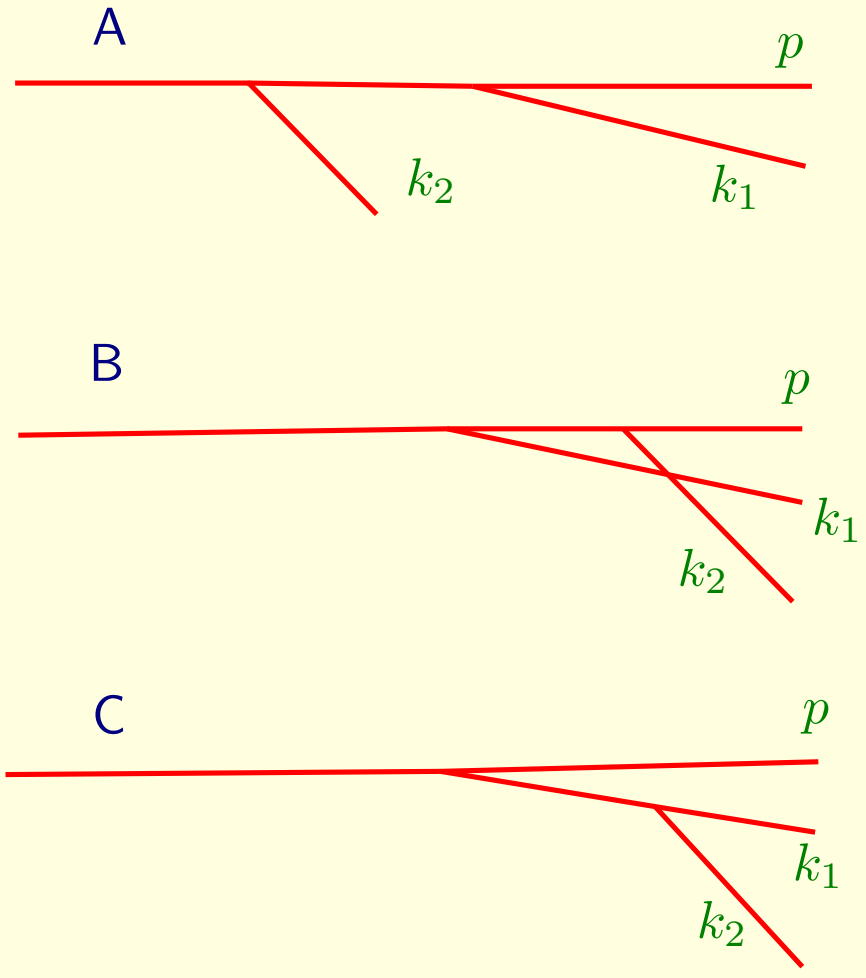
Intensity for photon jets $=0$

Intensity for gluon jets $=C_A$

instead of $2C_F + C_A$

Consistent with a boosted jet pair, in the case of a photon jet.
In angular ordered SMC large angle soft emission is generated first.
Hardest emission (i.e. highest pt) happens later.

Angular ordering is a non-trivial result; look at double emission



Largest angle gluon k_2 can be inserted in 3 ways;

$(k_2+p)^2 > (k_1+p)^2$:
only A contributes;
$(k_2+p)^2 < (k_1+p)^2$:
B+C contribute

In the last cases B and C add coherently: total as if k_2 was emitted in A, neglecting the virtuality of the incoming line.

Angular ordering found by Mueller in a 3-loop calculation of soft emissions

The theory of multiple soft emission has been extended from double log accuracy (i.e. only small angle) to large angle emissions. In the large N limit (only planar graphs) this formulation acquires the particularly simple form of an energy ordered dipole cascade.

Normally in soft gluon (and photon) physics, one uses the fact that the insertion of the softest gluon is IR divergent only if it is attatched to external lines. This is not the case if you have collinear singularities! (see previous slide) Dipole cascade is also non-trivial ...
(Fiorani, Marchesini, Reina, 1988)

Is coherence important?

- Eccessive multiplicity growth in virtuality ordered MC ("historical" problem with multiplicity when LEP was turned on)
- Angular ordered MC's (HERWIG) agree with mutliplicity data in e^+e^- annihilation
- Agreement of PYTHIA with multiplicity data was achieved by superimposing an angular ordered veto over the virtuality ordered shower. This amounts to take the interference as being totally destructive. No major differences between PYTHIA and HERWIG are seen if the angular order veto is applied.

So: if the MC does not include coherence correctly, approximate remedies should be found that adjust the main observables

k_T-clusters

Given a set of n particles in an e^+e^- final state, reconstruct jets by pairing up recursively pairs of particles with minimum

$$y_{kl} = 2(1-\cos\theta_{kl})\min\left(E_k^2, E_l^2\right)/Q^2.$$

The pair of particles with minimum y_{kl} are combined into a single pseudo-particle, with momentum $p_{kl} = p_k + p_l$ (or any variant of this, like the P or E_0 schemes). Notice:

$$y \approx \frac{p_T^2}{Q^2},$$

since

$$2(1-\cos\theta) \approx \theta^2, \quad \min\left(E_k, E_l\right) \approx \frac{E_k E_l}{E_k + E_l}\,.$$

k_T-cluster multiplicity can be computed at NLL level using the theory of multiple soft gluon emission (not possible for virtuality clusters)
In the following: how to reproduce the results of Catani, Dokshitzer, Olsson, Turnock and Webber, 1991 using angular ordering

k_T-clusters multiplicity calculation: use angular ordering!

Sudakov form factor as in angular ordered shower, but veto radiation that yields $y > y_{\min}$. Introducing: $Q_{\min} = \sqrt{y_{\min}}\, Q$, $t = \theta E$, $q = k_T = \sqrt{t}\, z(1-z)$

$$\Delta(Q) = \exp\left[-\int_0^{Q^2} \frac{dt}{t} \int dz\, \frac{\alpha_s(q)}{2\pi} P(z)\, \theta(q - Q_{\min}) \right]$$

$$= \exp\left[-2 \int \frac{dq}{q}\, dz\, \frac{\alpha_s(q)}{2\pi} P(z)\, \theta(q - Q_{\min})\, \theta(Q\, z(1-z) - q) \right].$$

For example, for P_{qq} (HOMEWORK PROBLEM!):

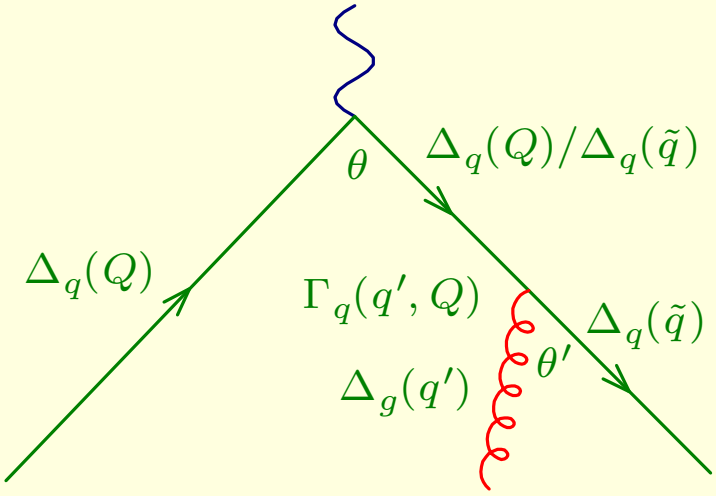
$$\Delta_q(Q) = \exp\left[-\int_{Q_{\min}}^{Q} \Gamma_q(q, Q)\, dq \right], \quad \Gamma_q(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log\frac{Q}{q} - \frac{3}{4} \right)$$

For P_{gg}, P_{gq}: $\Gamma_g(q, Q) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q} \left(\log\frac{Q}{q} - \frac{11}{12} \right)$, $\Gamma_f(q, Q) = \frac{N_F}{3\pi} \frac{\alpha_s(q)}{q}$

and $\Delta_g(Q) = \exp\left[-\int_{Q_{\min}}^{Q} [\Gamma_g(q, Q) + \Gamma_f(q, Q)]\, dq \right]$

Thus, the 2-clusters multiplicity is: $\frac{\sigma_2}{\sigma_{\rm tot}} = \Delta_q^2(Q)$.

3-clusters: The antiquark line gets a factor $\Delta_q(Q)$ as before.

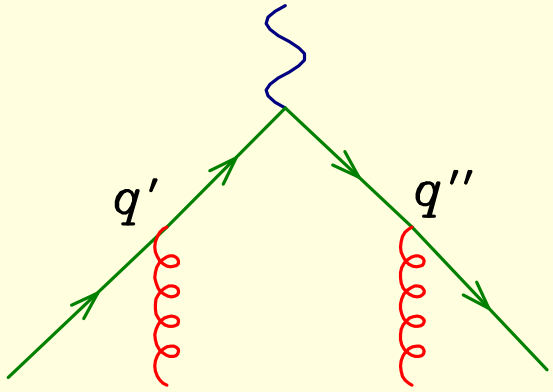


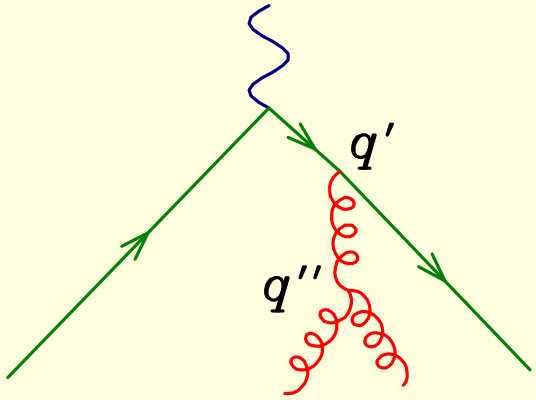
The gluon line from the gluon vertex gets a factor $\Delta_g(q')$, where $q' = \theta' E_g$.
(This uses angular ordering! no gluon radiation with angles $> \theta'$)
The quark line from the gluon vertex gets a factor $\Delta_q(\tilde{q})$, where $\tilde{q} = \theta' E_q$.
The quark line from the photon to the gluon vertex gets a factor:

$$\exp\left[-\int_{\theta'}^{\theta}\frac{d\theta^2}{\theta^2}\int dz\,\frac{\alpha_s(q)}{2\pi}P(z)\theta(q-Q_{\rm min})\right] \approx \frac{\Delta_q(Q)}{\Delta_q(\tilde{Q})}\ (\text{in the soft approximation!})$$

The gluon vertex gets a factor $\Gamma_q(Q')$.

Thus, the 3-clusters multiplicity is $2\Delta_q^2(Q)\int_{Q_{\rm min}}^{Q}\Delta_g(q')\Gamma_g(q',Q)\,dq'$





Same (angular ordering) arguments lead to the following values for the 4-cluster multiplicity diagrams:

$$\Gamma_q(q',Q)\Gamma_q(q'',Q)\,\Delta_{q\bar{q}\,gg}, \qquad \Gamma_q(q',Q)\Gamma_q(q'',q')\,\Delta_{q\bar{q}\,gg},$$

with $\Delta_{q\bar{q}\,gg} = \Delta_q^2(Q)\,\Delta_g(q')\,\Delta_g(q'')$.
The following observation holds to all orders: the Sudakov factors depend only upon the nodal values of the k_T scales q', q'', ... at which branching occours, and on the parton type.

The procedure works because, in an angular ordered shower, the starting evolution scale of a branched soft parton, $E_{\rm soft}\,\theta$, is equal to the k_T.

In CKKW: replace approximate Γ factors by exact matrix elements.

Detailed prescription:

- Consider the cross section $d\sigma_n$ to produce n partons ($n \leqslant N$), all separated by a minimum distance parameter $y_{\min}$, computed with a fixed value of α_s. Generate n and n body kinematics with probability $d\sigma_n$.
- From the given kinematics reconstruct the scheleton, by pairing up recursively partons with smallest y. Only pair up partons that can come from the same splitting process (i.e. gg, qg, $q\,\bar{q}$; no qq, $q'\,\bar{q}$, etc.). Assign to each vertex i of the skeleton the corresponding $q_i = Q\sqrt{y_i}$.
- Associate factors $\Delta(q_i)/\Delta(q_j)$ ($q_i > q_j$) with each intermediate line of the skeleton, a factor $\Delta(q_i)$ with each final line of the skeleton, and $\alpha_s(q_i)/\alpha_s(Q)$ with each node of the skeleton. Compute the product of all this factors and accept the event with a probability equal to this product.

Originally, N (and/or $Q_{\min}$) was assumed to be large enough, so that the result was insensitive to N (i.e., most events had less than N clusters)

Interfacing to a Shower

At this level, we must complete the calculation with a full shower. The calculation was performed by extending the Shower approximation with exact matrix elements, but only for splittings with k_T above $Q_{\rm min}$. In order to correctly extend the shower, we should:

A) Avoid to generate splittings with $k_T > Q_{\rm min}$; those were already generated by the matrix elements

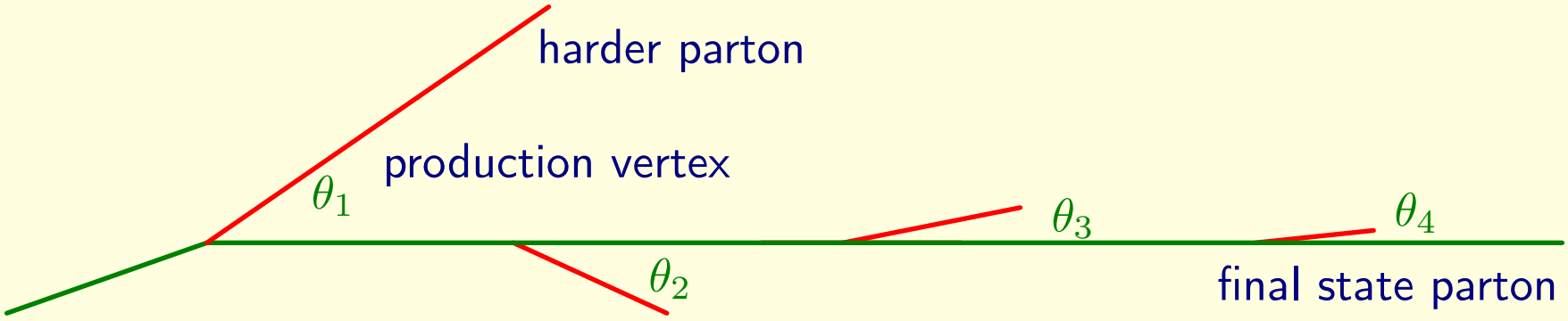
B) Include all missing radiation with $k_T < Q_{\rm min}$

Step A) is achieved by introducing a $\theta(Q_{\rm min} - k_T)$ in the splitting vertices and Sudakov form factors of the Shower Monte Carlo. In practice, this is achieved by the veto algorithm:

$\rightarrow$ At any stage of the generation of a branching starting from a scale t' in the SMC, generate the branching at a scale $t'' < t'$ and generate the z value with the usual method.

$\rightarrow$ If $k_T = \sqrt{t} z(1-z) > Q_{\rm min}$, discard the current branching, set t' to the value t'', and go back to the previous step. Otherwise, continue.

Step B) is more subtle: one should allow branchings from each intermediate and final line of the skeleton that were not included in the ME calculation.
The angular ordered SMC should introduce, for any intermediate line initiating at an angle θ' and ending at an angle θ'', radiation with $\theta' > \theta > \theta''$, for any k_T. But only $k_T > Q_{\min}$ was provided by the ME.
Thus, a truncated shower (P.N. 2004), with the angular ordered bound $\theta' > \theta > \theta''$, and a k_T veto $k_T < Q_{\min}$ (as before) should be provided for each intermediate line.
For final state line, a standard vetoed shower (i.e. $\theta' > \theta$, unconstrained from below), should be provided.
CKKW proposed an almost equivalent solution to this problem:
The final state particle are fed into an angular ordered Monte Carlo, their initial showering angle is set equal to the angle at the vertex where the parton was initially produced.
The vertex where the parton is initially produced is found by walking up from the given final state parton in the shower skeleton, skipping vertices where the parton in question is merged with a softer parton, and stopping at the first vertex where this is not the case.

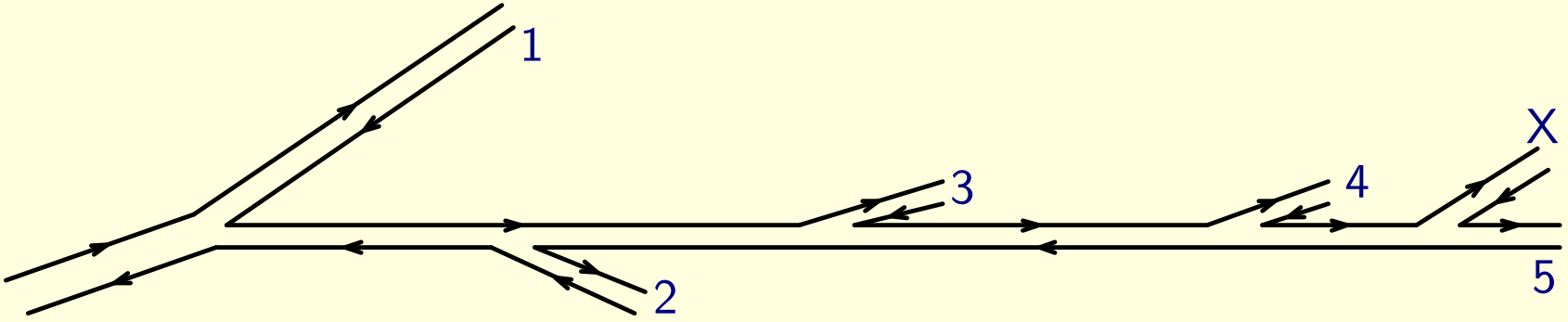


The CKKW prescription provides a single shower from θ_1. The green line from θ_1 has basically constant energy, since radiation from 2, 3, 4 is soft. So, a shower from θ_1 to the minimum is like a shower from θ_1 to θ_2, plus a shower from θ_2 to θ_3 plus a shower from θ_4 to the minimum.
It is then obvious that the CKKW prescription is equivalent, from a kinematical view point, to add a truncated shower to all internal skeleton lines.

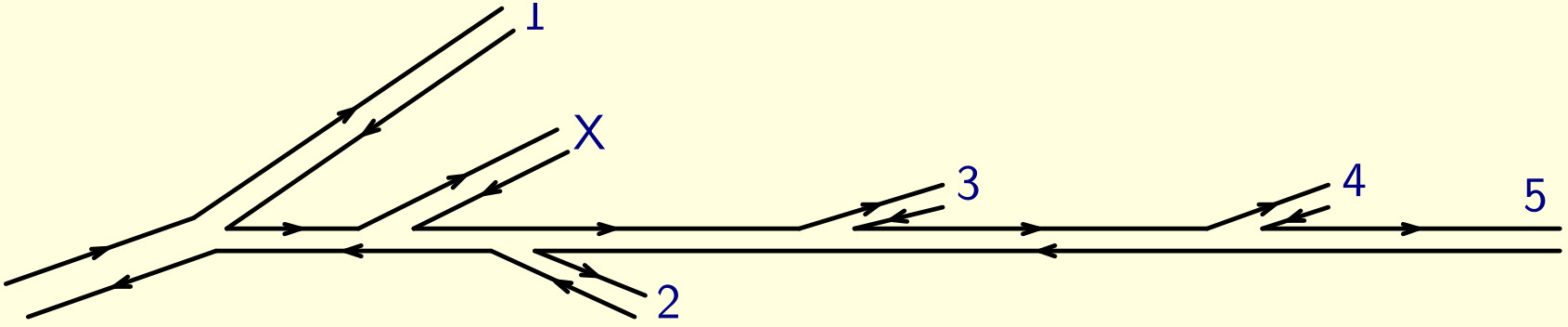
HOWEVER: colour pattern wrong ...

Comparison of colour connections for CKKW and truncated showers

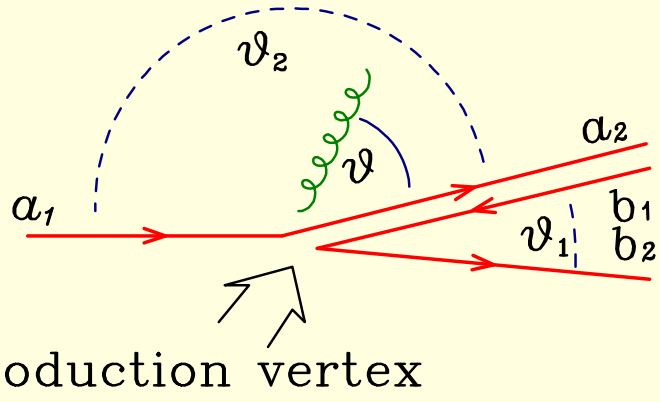
Consider the emission of parton X with $\theta_1 > \theta > \theta_2$.
Colour connection in CKKW:



colour connection with truncated shower:



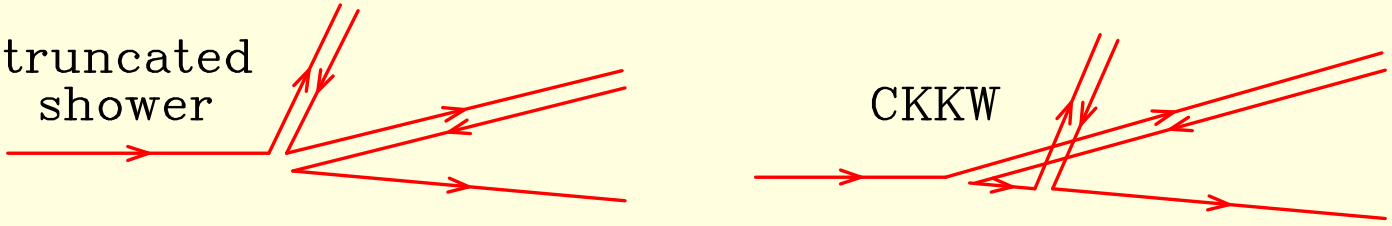
X is close in colour to 4 and 5 in CKKW, to 1 and 3 with truncated showers:
Larger colour gaps with CKKW



Consider $e^+e^- \to q\,\bar{q}\,g$.
Assume θ_1 small. Consider gluon emission with angle $\theta \gg \theta_1$, $\theta \ll \theta_2$.
Coherence requires that the emission strength is C_F (gluon and quark coherently)

In HERWIG: initial angle for gluon radiation is θ_1 or θ_2 with a 50% probability. Thus (in the above region) strength is $C_A/2 \approx C_F$ (but only in the average!!)

In CKKW: radiation from gluon restricted to $\theta < \theta_1$, quark radiates with angle up to θ_2. Thus only the quark radiates in the above region, with strength C_F. However, the colour connection is incorrect! Large colour gap ...



So: coherent showers are always needed when doing ME-Shower matching with angular ordered showers.

CKKW with finite N

In the original CKKW scheme, N is assumed to be large enough (i.e., almost negiglible amount of final states with N clusters). Since N is practiccally finite, this means that $Q_{\min}$ should be kept large enough.
A practical alternative to this (Mrenna and Richardson, 2003; Schaelicke and Krauss, 2005) is the following:

In the matrix element for N clusters, replace the $Q_{\min}$ scale used to compute the Sudakov form factors and the vetoed showers with Q_n, (the $\sqrt{y}Q$ value of the smallest cluster.)

This was, the parton shower will be able to generate $N+1$, $N+2$, etc. clusters with merging scales larger than $Q_{\min}$, but the N hardest pairings will be accurate at the matrix element level, while the subsequent ones will be only collinear accurate.

Notice: with this prescription $Q_{\rm min}$ can be chosen as low as one likes (i.e., even near the shower cutoff). In this limiting case, no subsequent showers will be generated by the Monte Carlo for events with less than N clusters.

In all cases, the scale $Q_{\rm min}$ and N appear here as the delimiter between the exact matrix element calculation and the shower approach: production of more than N clusters will rely upon the SMC, as well as production of clusters below $Q_{\rm min}$.

Summary of CKKW

- Provides smooth interface between ME and Shower
- Uses a separation scale $Q_{\rm min}$, but is actually not strictly necessary
- Treats consistently multiple soft emissions in QCD, including interference effects (i.e., it is fully consistent with angular ordering)
- Cancellation of $Q_{\rm min}$ dependence is demonstrated, provided the SMC is of the angular ordered type (i.e. HERWIG like)

In practice, its use has been extended also to SMC of different kind (virtuality ordered, dipoles with k_T ordering, etc.)
If the SMC treats correctly coherence of multilpe soft gluon emission, it should be possible to interface it into a CKKW scheme preserving this accuracy.
If not, only soft emissions above $Q_{\rm min}$, and in number $\leqslant N$, will be correct in the soft limit.

Variants

Several alternatives have been proposed:

- MLM matching (ALPGEN group)
- Pseudo showers (Mrenna and Richardson, 2003)
- CKKW-Lönnblad (Lönnblad, 2002)

mostly to avoid computing explicitly the Sudakov form factors;

It would be interesting to discuss in details the relation of these methods with CKKW.

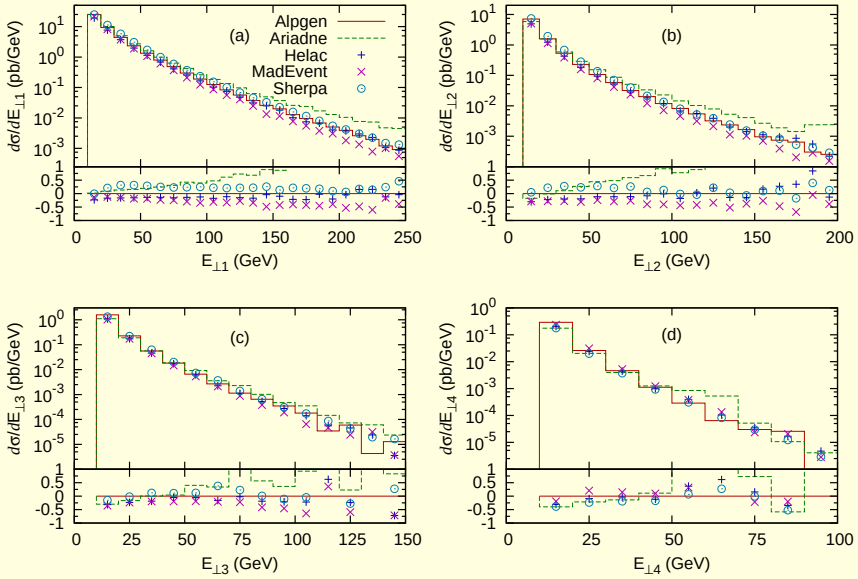
Many approaches introduce variants of the procedure to remedy to the problems created by the imperfect matching.
A critical comparison of the various methods is outside the scope of this lecture ... However

Comparison among different ME generators

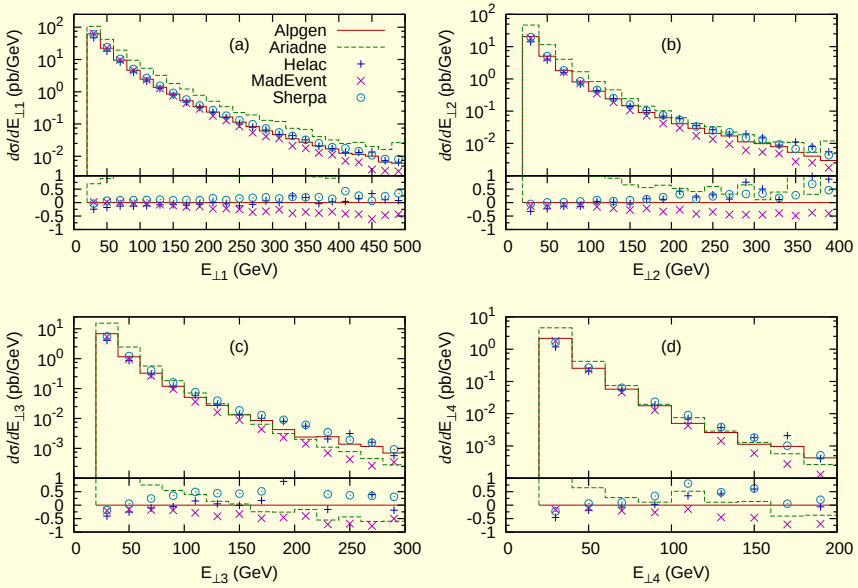
(Alwall etal, Jul.07): compare Alpgen,Ariadne,Helac,MadEvent,Sherpa

$W + n\,\text{jets}$, jet E_T spectra

TEVATRON



LHC



THE MESSAGE:

good agreement among different ME implementation, in spite of different matching prescriptions (CKKW, MLM, and others)

Some bibliography

- Catani, Krauss, Kuhn, Webber, JHEP 11 (2001) 063, hep-ph/01092231
- Catani, Dokshitzer, Olsson, Turnock, Webber, Phys.Lett.B269(1991)432 k_T algoritm
- Mueller, Phys. Lett B104(1981)161, Angular ordering, plus several other: Ermolaev and Fadin, Bassetto, Ciafaloni, Marchesini, etc.
- Mrenna and Richardson, JHEP 05 (2002) 046, hep-ph/0312274 (pseudo shower implementation of CKKW)
- Schaelicke and Krauss, hep-ph/0503281 (SHERPA implementation)
- Lönnblad, JHEP 05(2002)046, hep-ph/0109231 (CKKW-L)
- Lavesson and Lönnblad, arXiv:0712.2966, comparison of various methods in e^+e^-
- P.N., JHEP0411:040,2004,hep-ph/0409146 (POWHEG, trunc. showers)
- Hoeche, Krauss, Schumann, Siegert, JHEP 0905:053,2009 arXiv:0903.1219, on truncated showers

NLO Calculations

SMC with ME-corrections are only leading order accurate. Scale uncertainty

$$\alpha_s^n(2\mu) \approx \alpha_s^n(\mu)(1 - b_0\alpha_s(\mu)\log(4))^n \approx \alpha_s(\mu)(1 - n\alpha_s(\mu))$$

For $\mu = 100\,\mathrm{GeV}$, $\alpha_s = 0.12$;

	$W+1J$	$W+2J$	$W+3J$
uncertainty:	$\pm 12\%$	$\pm 24\%$	$\pm 36\%$

This scale uncertainty can be considered as an estimate of the error due to missing higher order terms

To improve on this, need to go to NLO

Positive experience with NLO calculations at LEP, HERA, Tevatron (we TRUST perturbative QCD after LEP!).

Huge NLO effort for the computation of signals and backgrounds for LHC.

LHC priority wish list, Les Houches 2005 (hep-ph/0604120)

process, $V \in \gamma, W^{\pm}, Z$	background to	As of now
$pp \to VV + 1j$	$t\bar{t}H$, BSM	WW
$pp \to H + 2j$		*
$pp \to t\bar{t} + b\bar{b}$	$t\bar{t}H$	New!
$pp \to t\bar{t} + 2j$	$t\bar{t}H$	$t\bar{t} + 1j$ (2007)
$pp \to VV + b\bar{b}$	VBF $\to VV$, $t\bar{t}H$, BSM	*
$pp \to VV + 2j$	VBF $\to VV$	*
$pp \to V + 3j$	BSM signatures	New!
$pp \to VVV$	SUSY trilepton	ZZZ, WWZ
$pp \to b\bar{b}b\bar{b}$	Higgs and BSM	

Recent contributions:
$W + 3\,j$: Ellis, Melnikov, Zanderighi 2009; Berger etal, 2009;
$pp \to t\bar{t} + b\bar{b}$: Bredstein, Denner, Dittmaier, Pozzorini, 2009;

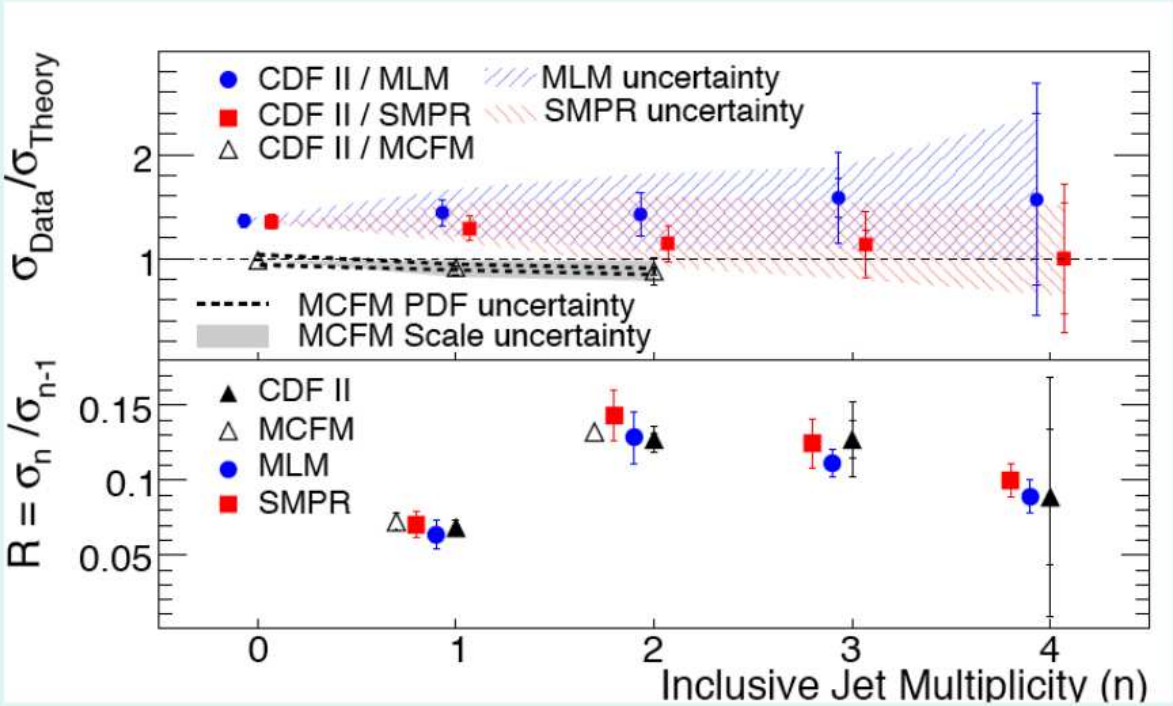
Unlike tree level processes, research groups still focus upon specific processes;

However, very complex calculations (like $W \to 3\,\mathrm{partons}$ at NLO) are possible.

Special techniques to compute loop graphs are needed;

In particular, a technique by Ossola, Papadopoulos and Pittau (2007) leads to hope that full automation of these calculations will become soon a reality,

W+n jet rates from CDF

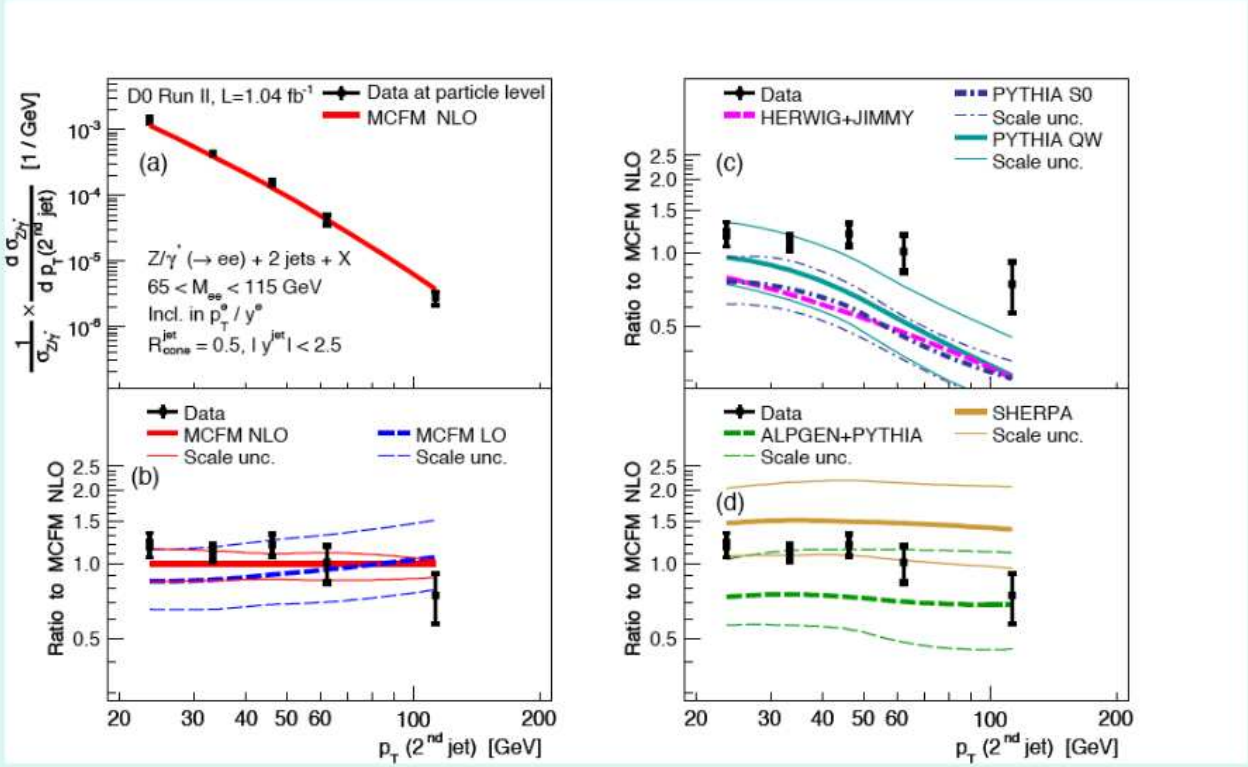


Both uncertainty on rates and deviation of Data/Theory from 1 are smaller than other calculations. The ratio R agrees well for all theory calculations, but only available with from MCFM with small error for n≤2.

Keith Ellis, Madison 2009

MCFM RKE, Campbell

New Z + jets results from D0



- MCFM, LO and NLO agrees with data;
- shower-based generators show significant differences with data;
- matrix element + parton shower models agree in shape, but with larger normalization uncertainties.

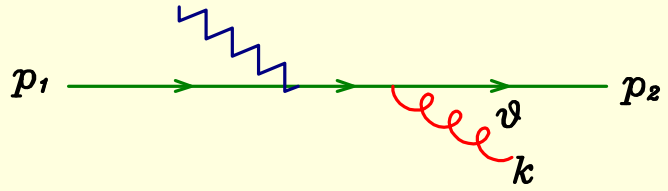
So: NLO calculations represent well the data;

But: NLO results are cumbersome and unfriendly: typically made up of an n-body (Born+Virtual+Soft and Collinear remnants) and $n+1$ body (real emission) terms, both divergent (finite only when summed up).

The same problems that we find with ME results are made worse when NLO corrections are included.

Simple example: Z production

"Real" contribution to $q\bar{q} \to Z + X$:
(in fact, real + divergent part of virtual)



$$\frac{C_F}{N_c}\frac{g_Z^2 g_s^2}{32\pi^2}\frac{1}{S}\left[2(1+y^2)\xi^2+8(1-\xi)\right]\left\{\frac{1}{2}\left(\frac{1}{\xi}\right)_+\left[\left(\frac{1}{1-y}\right)_+ + \left(\frac{1}{1-y}\right)_-\right]\right\} d\xi\, dy\, dY_Z$$

where

- Y_Z is the Z rapidity
- $y = \cos\theta$, θ being the emission angle of the gluon in the partonic CM
- $\xi = 2k^0/\sqrt{s}$ in the partonic CM ($s = (p_1 + p_2)^2$)

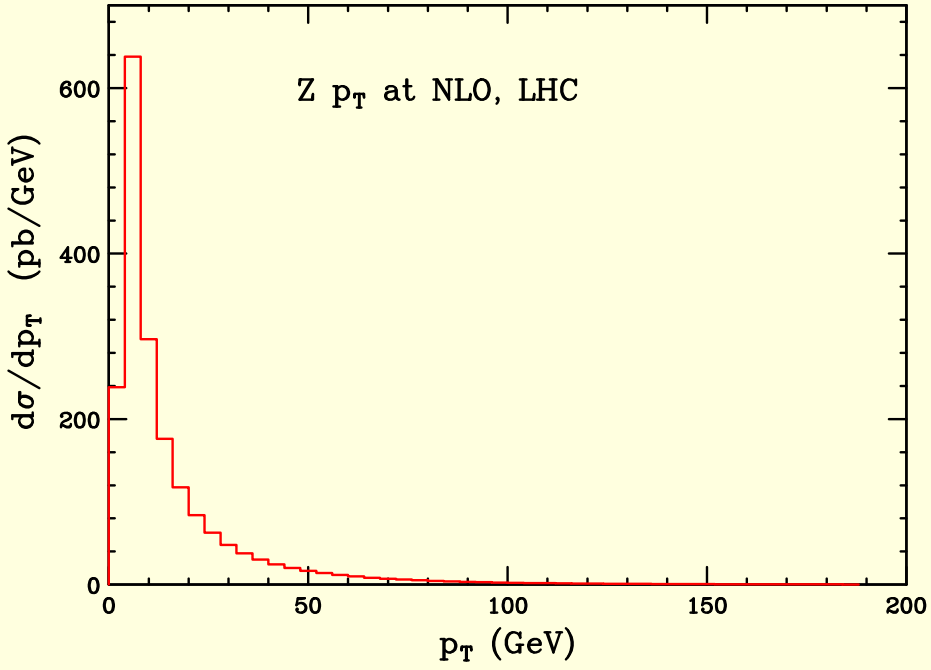
$$\left(\frac{1}{\xi}\right)_+ = \lim_{\epsilon\to 0}\left[\frac{1}{\xi+\epsilon} - \log\frac{1}{\epsilon}\,\delta(\xi)\right];\ \int_0^1 \left(\frac{1}{\xi}\right)_+ = 0;\ \int_{-1}^1 \left(\frac{1}{1\pm y}\right)_+ = 0,$$

Notice: $p_T^Z = \frac{\sqrt{s}}{2}\sqrt{1-y^2}\,\xi$.

Divergent contributions to the cross section for $p_T^Z > 0$ (i.e. $\xi > 0$, $1 \pm y > 0$), compensated by negative divergences (i.e. $\delta(\xi)$, $\delta(1 \pm y)$ terms) at $p_T^Z = 0$, that arise from the virtual corrections.

p_T^Z at NLO:

For small enough histogram bins the first bin will always turn negative!



A negative bin means: $\mathcal{O}(\alpha_s)$ corrections larger than Born term:
cannot trust perturbation theory!
One should carefully decide the appropriate bin size around the origin.
For more complex processes this becomes a requirement on jet parameters.

To get a finite cross section we must define our Z cross section allowing jets with $p_T < M$. If M is too small, the cross section turns negative!

Remember the lecture on ME: corrections like $\text{Born} \times \left(1 - \alpha_s \log \frac{Q}{M}\right)$

For more complex processes, we need a jet parameter M, and a jet definition. For example, we may build clusters recursively until the cluster mass stays below M, and require $p_T > M$ for our jets.

M should be carefully chosen, as shown above.

So: NLO calculations represent well the data;
Can we MERGE NLO and Showers?
Some sort of resummation of the diverging virtual corrections should be carried out, in order to get sensible results in the dangerous regions of collinear and soft emissions.

The key to the solution: the dangerous region is well described by the factorization formula. For example, for $y \to 1$ our cross section becomes

$$\frac{C_F}{N_c}\frac{g_Z^2\, g_s^2}{16\pi^2}\frac{1}{S}\left[\frac{x^2+1}{(1-x)_+}\right]\frac{dy}{1-y}\; d\xi\, dy\, dY_W\,, \quad \text{with}\, x = 1-\xi$$

The problem of diverging negative virtual corrections is dealt with and solved in the Shower formalism.

In the following: assume that the hardest SMC radiation is the first one, i.e. that the Shower is ordered in relative p_T. We deal later with the subtle issue on the choice of the ordering variable.

Look back at the cross section for the first emission in a Shower Monte Carlo

$$d\sigma = d\Phi_B\, B(\Phi_B)\left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \sum_{(jk)} \underbrace{\Delta_{t_I,t}\ \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially, t_0 is the minimum hardness of emission
- $\Delta_{t_I,t}$ is the no-radiation probability with hardness $> t$

$$\Delta_i(t_I,t) = \exp\left[-\sum_{(jk)} \int_t^{t_I} \frac{dt'}{t'} \int dz\, \frac{\alpha_s(t')}{2\pi}\, P_{i,jk}(z) \right]$$

Expand the Shower formula at order $\mathcal{O}(\alpha_s)$:

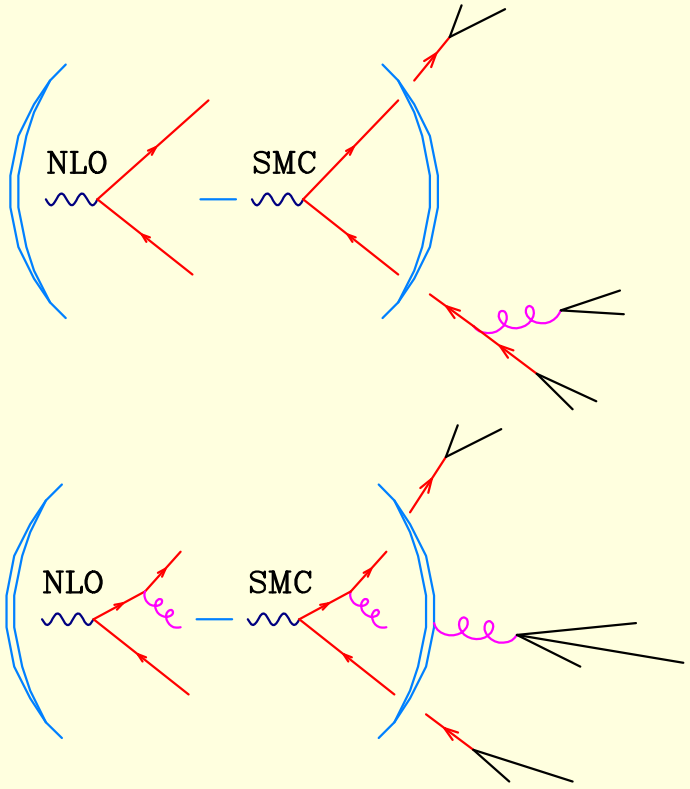
$$d\sigma = d\Phi_B\, B(\Phi_B)\left(1 - \underbrace{\sum_{(jk)} \int_{t_0}^{t_I} \frac{dt'}{t'} \int dz\, \frac{\alpha_s}{2\pi}\, P_{i,jk}(z)}_{\text{virtual}} + \sum_{(jk)} \underbrace{\frac{\alpha_s}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}}_{\text{real}} \right)$$

As in the NLO calculation, we have a negative divergent contribution for no radiation, and a positive divergent contribution for radiation.
The divergence cancels for inclusive cross sections.

So: the SMC has his own approximate NLO virtual and real terms. To get NLO accuracy these terms should be modified to yield the exact NLO.

Notice that SMC algorithms reconstruct from Born kinematics Φ_B
and radiation variables t, z, ϕ, the full phase space Φ (momentum reshuffling)

MC@NLO (2002, Frixione+Webber)



Add difference between exact NLO and approximate (MC) NLO.

- Must use MC kinematics
- Difference should be regular (if the MC is OK)
- Difference may be negative

Several collider processes already there:
Vector Bosons, Vector Bosons pairs, Higgs, Single Top (also with W), Heavy Quarks, Higgs+W/Z.

How it works (roughly)

The cross section for the hardest event in MC@NLO is

$$d\sigma = \underbrace{\bar{B}^{\rm MC}(\Phi_B) d\Phi_B}_{S\,\text{event}} \left[\underbrace{\Delta^{\rm MC}_{t_0} + \Delta^{\rm MC}_t \frac{R^{\rm MC}(\Phi)}{B(\Phi_B)} d\Phi^{\rm MC}_r}_{\text{MC shower}} \right] + \left[\underbrace{R(\Phi) - R^{\rm MC}(\Phi)}_{H\,\text{event}} \right] d\Phi$$

$$\bar{B}^{\rm MC}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^{\rm MC}(\Phi)\, d\Phi^{\rm MC}_r}_{\text{infinite}} \right]}_{\text{finite}}$$

Imagine that soft and collinear singularities in $R^{\rm MC}$ are regulated as in V.

The full phase space Φ is parametrized in terms of the Born phase space Φ_B and the radiation variables of the MC $\Phi^{\rm MC}_r$ (typically z, t, ϕ), according to the MC procedure (reshuffling) that yields Φ from Φ_B and $\Phi^{\rm MC}_r$.

B : Born cross section; V : exact virtual cross section.

$R^{\rm MC}$: radiation cross section in the MC, typically: $R^{\rm MC} = B\,\frac{1}{t}\,\frac{\alpha}{2\pi} P(z)$

R : exact radiation cross section;

We can check that the $\mathcal{O}(\alpha_s)$ expansion of $d\sigma$ coincides with the exact NLO;

$$d\sigma = \bar{B}^{\mathrm{MC}}(\Phi_B) d\Phi_B \left[\Delta_{t_0}^{\mathrm{MC}} + \Delta_t^{\mathrm{MC}} \frac{R^{\mathrm{MC}}(\Phi)}{B(\Phi_B)} d\Phi_r^{\mathrm{MC}} \right] + [R(\Phi) - R^{\mathrm{MC}}(\Phi)] d\Phi$$

$$\bar{B}^{\mathrm{MC}}(\Phi_B) = B(\Phi_B) + \left[V(\Phi_B) + \int R^{\mathrm{MC}}(\Phi)\, d\Phi_r^{\mathrm{MC}} \right]$$

Expand:

$$d\sigma = \left[B + V + \int R^{\mathrm{MC}} d\Phi_r^{\mathrm{MC}} \right] d\Phi_B \left[1 - \int \frac{R^{\mathrm{MC}}}{B}\, d\Phi_r^{\mathrm{MC}} + \frac{R^{\mathrm{MC}}}{B} d\Phi_r^{\mathrm{MC}} \right] + [R - R^{\mathrm{MC}}] d\Phi$$

$$= [B + V] d\Phi_B + B d\Phi_B \left[\int \frac{R^{\mathrm{MC}}}{B} d\Phi_r^{\mathrm{MC}} - \int \frac{R^{\mathrm{MC}}}{B}\, d\Phi_r^{\mathrm{MC}} + \frac{R^{\mathrm{MC}}}{B} d\Phi_r^{\mathrm{MC}} \right] + [R - R^{\mathrm{MC}}]\, d\Phi$$

$$= [B + V] d\Phi_B + B d\Phi_B \left[\frac{R^{\mathrm{MC}}}{B} d\Phi_r^{\mathrm{MC}} \right] + [R - R^{\mathrm{MC}}]\, d\Phi = [B + V] d\Phi_B + R d\Phi$$

Recipe for MC@NLO

- Compute totals for S and H events:

$$\sigma_S = \int |\bar{B}^{\rm MC}(\Phi_B)| d\Phi_B, \;\; \sigma_H = \int |R - R^{\rm MC}| d\Phi$$

- Chose an S or H event with probability proportional to σ_S, σ_H
- For an S event:
  - generate Born kinematics with probability

$$|\bar{B}^{\rm MC}(\Phi_B)| = \left| B(\Phi_B) + \left[V(\Phi_B) + \int R^{\rm MC}(\Phi)\, d\Phi_r^{\rm MC} \right] \right|$$

  - Feed the Born kinematics to the MC for subsequent shower with weight ± 1, same sign as $\bar{B}^{\rm MC}(\Phi_B)$.
- For an H event:
  - generate Radiation kinematics with probability $|R - R^{\rm MC}|$.
  - Feed to the MC (with weight ± 1, same sign as $R - R^{\rm MC}$)

Issues:

- Must use of the MC kinematic mapping $(\Phi_B, \Phi_r^{\rm MC}) \Rightarrow \Phi$.
- $R - R^{\rm MC}$ must be non singular: the MC must reproduce exactly the soft and collinear singularities of the radiation matrix element. (Many MC are not accurate in the soft limit)
- The cancellation of divergences in the expression of $\bar{B}^{\rm MC}$ is taken care of in the framework of the subtraction method (cancellation of divergences under the integral sign) so that the integral in $\bar{B}^{\rm MC}$ becomes in fact convergent.
- Negative weights in the output (not like standard MC's).

POWHEG

Positive Weight Hardest Emission Generator

Method to generate the hardest emission first, with NLO accuracy, and independently of the SMC (P.N. 2004).

- SMC independent; no need of SMC expert; same calculation can be interfaced to several SMC programs with no extra effort
- SMC inaccuracies in the soft region only affect next-to-hardest emissions; no matching problems
- As the name says, it generates events with positive weight

How it works (roughly)

In words: works like a standard Shower MC for the hardest radiation, with care to maintain higher accuracy.

In a standard MC, the hardest radiation cross section is

$$d\sigma = d\Phi_B\, B(\Phi_B)\left(\underbrace{\Delta_{t_I,t_0}}_{\text{No radiation}} + \underbrace{\Delta_{t_I,t}\ \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\,\frac{dt}{t}\,dz\,\frac{d\phi}{2\pi}}_{\text{radiation}} \right)$$

- t_I is the maximum hardness allowed initially
- $\Delta_{t_I,t}$ in the no-radiation probability with hardness $> t$

SMC algorithm reconstructs from Born kinematics Φ_B and radiation variables t, z, ϕ, the full radiation phase space Φ (momentum reshuffling)

We say that Φ_B is the underlying Born configuration of Φ according to the mapping defined by the MC algorithm

Steps to go NLO:

$$(\Phi_B, t, z, \phi) \Leftrightarrow \Phi \qquad \Longrightarrow \quad (\Phi_B, \Phi_r) \Leftrightarrow \Phi,\ d\Phi = d\Phi_B\, d\Phi_r$$

$$B(\Phi_B) \qquad \Longrightarrow \quad \bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \overbrace{\int R(\Phi_B, \Phi_r)\, d\Phi_r}^{\text{INFINITE}} \right]}_{\text{FINITE!}}$$

$$\frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi} \quad \Longrightarrow \quad \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)}\, d\Phi_{\text{rad}}$$

POWHEG cross section:

$$d\sigma = d\Phi_B \bar{B}(\Phi_B) \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \qquad \Delta_t = \exp\left[\underbrace{-\int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r}_{\text{FINITE because of } \theta \text{ function}} \right]$$

with $t_r = k_T(\Phi_B, \Phi_r)$, the transverse momentum for the radiation.
In the collinear limit, k_t^2 must be of the order of t.

How does it work: $d\sigma = d\Phi_{_B}\, \bar{B}(\Phi_{_B}) \left[\Delta_{t_0} + \Delta_t \dfrac{R(\Phi)}{B(\Phi_{_B})} d\Phi_r \right],$

For small k_T, the factorization theorem yields

$$\frac{R(\Phi)}{B(\Phi_{_B})}\, d\Phi_{\rm rad} \approx \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z)\, \frac{dt}{t}\, dz\, \frac{d\phi}{2\pi}$$

and

$$\bar{B} \approx B \times (1 + \mathcal{O}(\alpha_s))$$

Thus: all features of SMC's are preserved at small k_T.
For large k_T, $\Delta \to 1$,

$$d\sigma = \bar{B} \times \frac{R}{B} \approx R \times (1 + \mathcal{O}(\alpha_s)),$$

so large k_t accuracy is preserved. Integrating in $d\Phi_r$ at fixed $\Phi_{_B}$

$$\int \delta(\Phi_{_B} - \bar{\Phi}_{_B})\, d\sigma = \bar{B}(\bar{\Phi}_{_B})$$

So NLO accuracy is preserved for inclusive quantities.

Example of mapping $\Phi \Leftrightarrow (\Phi_B, \Phi_r)$: Z pair production

Φ_B variables: choose $M_{\rm ZZ}$, $Y_{\rm ZZ}$ and θ, where

- $M_{\rm ZZ}$: invariant mass of the $Z\,Z$ pair
- $Y_{\rm ZZ}$: rapidity of $Z\,Z$ pair
- θ: go in the (longitudinally) boosted frame where $Y_{\rm ZZ} = 0$.
  go to the $Z\,Z$ rest frame with a transverse boost
  In this frame θ is the angle of a Z to the longitudinal direction.

Φ_r variables:

- $x = M_{\rm ZZ}/s$, (s is the invariant mass of the incoming parton system)
  $x \to 1$ is the soft limit
- y: cosine of the angle of the radiated parton to the beam direction
  in the partonic CM frame.
- ϕ: radiation azimuth.

Few tricks

Both in MC@NLO and POWHEG, integrals of the form

$$\bar{B}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\overbrace{V(\Phi_B)}^{\text{INFINITE}} + \overbrace{\int R(\Phi_B, \Phi_r)\, d\Phi_r}^{\text{INFINITE}}\right]}_{\text{FINITE!}}$$

are expressed within the subtraction method as

$$\bar{B}(\Phi_B) = B(\Phi_B) + V_{\rm SV}(\Phi_B) + \int d\Phi_r \left[R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r)\right]$$

Needs one Φ_r integrations for each Φ point!. To overcome this, we write

$$\tilde{B}(\Phi_B, \Phi_r) = \frac{B(\Phi_B) + V(\Phi_B)}{\int d\Phi_r} + R(\Phi_B, \Phi_r) - C(\Phi_B, \Phi_r)\,, \bar{B}(\Phi_B) = \int \tilde{B}(\Phi_B, \Phi_r) d\Phi_r\,.$$

so that

$$\bar{B}(\Phi_B \Phi) = \int \tilde{B}(\Phi_B \Phi, \Phi_r) d\Phi_r\,.$$

Use standard procedures (SPRING-BASES, Kawabata; MINT, P.N.)
to generate unweighted events for $\tilde{B}(\bar{\Phi}, \Phi_r) d\Phi_r d\bar{\Phi}$,
discard Φ_r (same as integrating over it!).

Radiation in POWHEG: $\Delta(\Phi_B, p_T) = \exp\left[-\int \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} \theta(k_T(\Phi_B, \Phi_r) - p_T) d\Phi_r \right],$

Look for an upper bounding function;

$$\frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} \leq U(\Phi) = N \frac{\alpha(k_T)}{(1-x)(1-y^2)}$$

Generate x, y according to

$$\exp\left[-\int U(\Phi_B) \theta(k_T(\Phi_B, \Phi_r) - p_T) d\Phi_r \right]$$

accept the event with a probability

$$\frac{R(\Phi_B, \Phi_r)}{B(\Phi_B) U(\Phi_B)} .$$

If the event is rejected generate a new one for smaller p_T, and so on (Veto method)

POWHEG: Interfacing to SMC's

For a p_T ordered SMC, nothing else needs to be done.
Use the standard Les Houches Interface for User's Processes (LHI):
put partonic event generated by POWHEG on the LHI;
Run the SMC in the LHI mode.
The LHI provides a facility to pass the p_T of the event to the SMC (SCALUP).
As far as the hardest emission is concerned, POWHEG can reach:

- NLO accuracy of (integrated) shape variables
- Collinear, double-log, soft (large N_c) accuracy of the Sudakov FF.
  (In fact, corrections that exponentiates are obviously OK)

As far as subsequent (less hard) emissions, the output has the accuracy of the SMC one is using.

Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \mathrm{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006),
  $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \to Z + \mathrm{jet}$, Very preliminary (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, Very preliminary, (Alioli, Oleari, Re, P.N., 2009)

In practice

MC@NLO: Code and manuals at

http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/

1 program for all processes

POWHEG: Codes and manuals in

http://moby.mib.infn.it/~nason/POWHEG

Examples are provide to link POWHEG to HERWIG or PYTHIA,

or to generate a Les Houches Event File to be fed

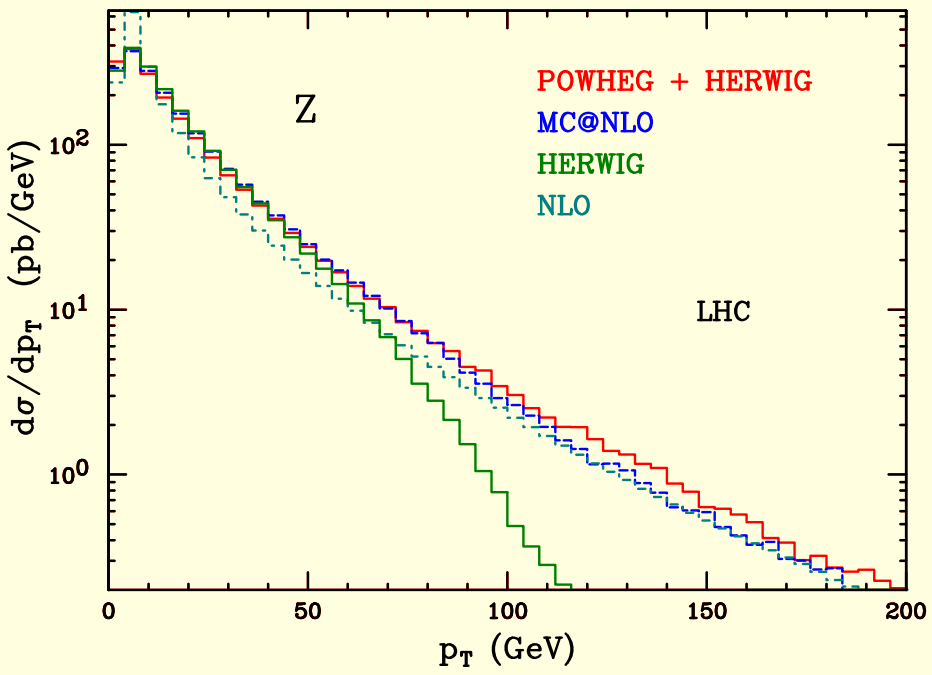
later to a SMC for showering.

1 program for each process

In the HERWIG++ code there are few independent implementations of MC@NLO and POWHEG processes

Examples: Z production

HERWIG alone fails ar large p_T;
NLO alone fails at small p_T;
MC@NLO and POWHEG work
in both regions;
Notice:
HERWIG with ME corrections
or any ME program, give the
same NLO shape at large p_T
However: Normalization around
small p_T region is incorrect
(i.e. only LO).



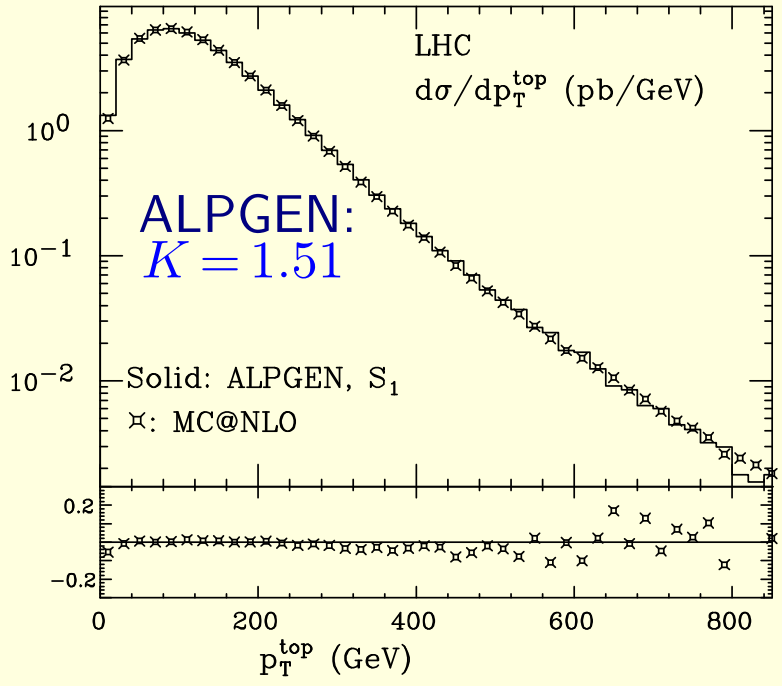
The essence of the improvement with respect to standard shower and ME matched programs is summarized in this plot.
Be careful with the misleading language: Z at LO $\mathcal{O}(1)$, NLO $\mathcal{O}(\alpha_s)$;
At $\mathcal{O}(1)$ there is no Z transverse momentum. Thus, the p_T distribution $p_T > 0$ is of $\mathcal{O}(\alpha_s)$, i.e. has leading order accuracy!

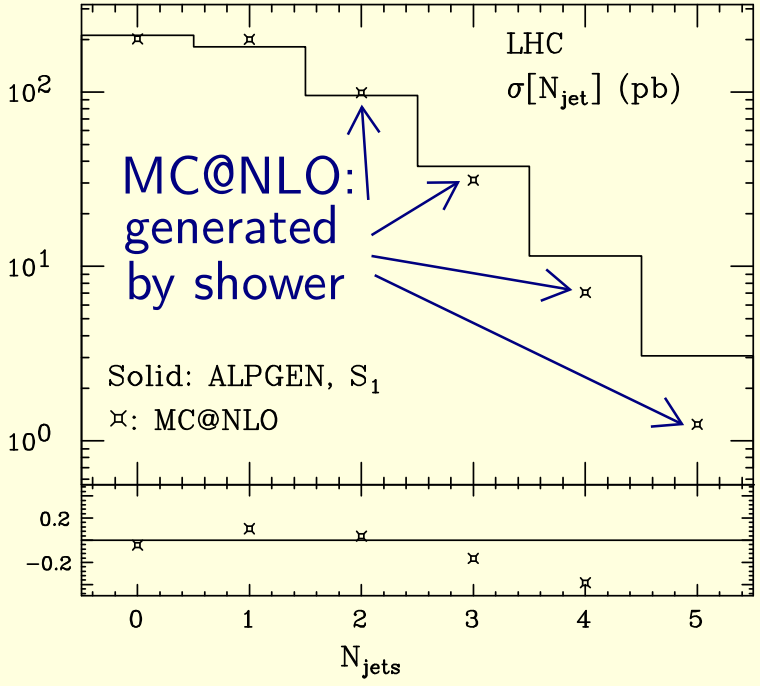
NLO+PS compared with ME programs: ALPGEN and MC@NLO in $t\bar{t}$ production

expect:

- Disadvantage: worse normalization (no NLO)
- Advantage: better high jet multiplicities (exact ME)

(Mangano, Moretti,Piccinini,Treccani, Nov.06)



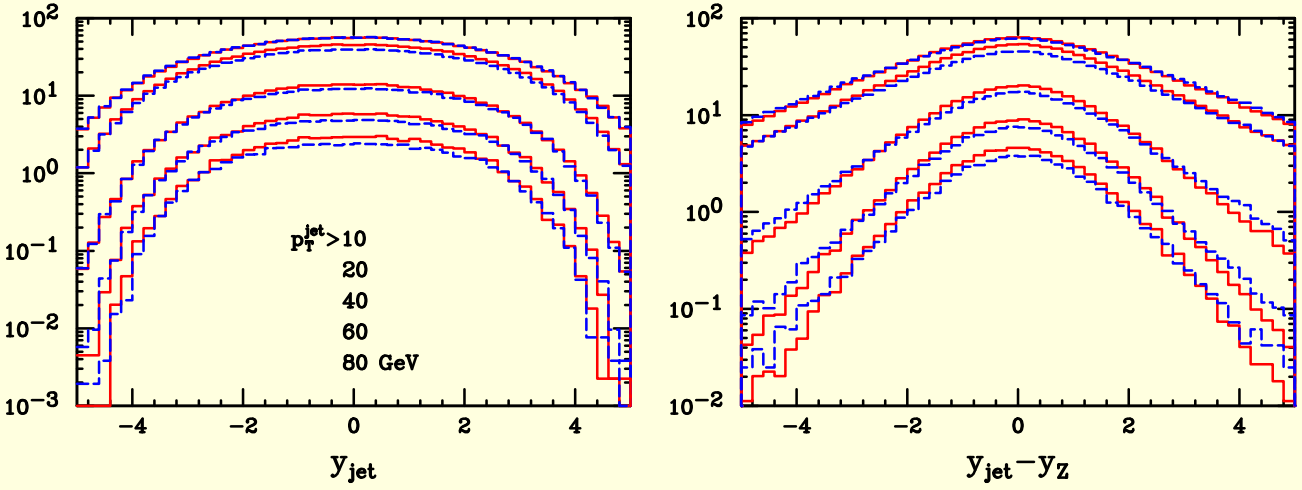


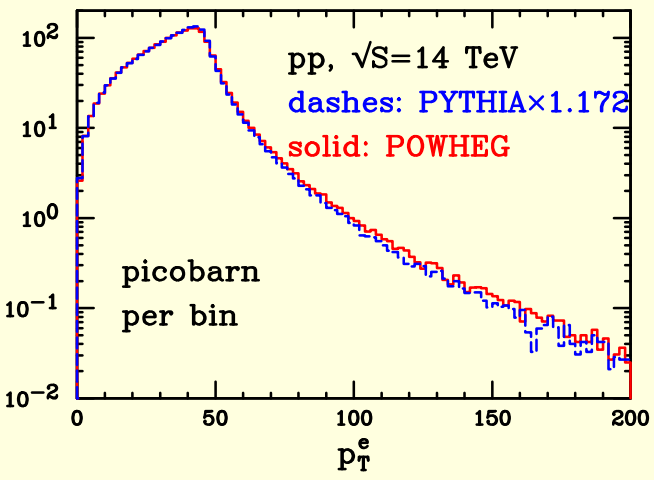
PYTHIA ME vs. POWHEG

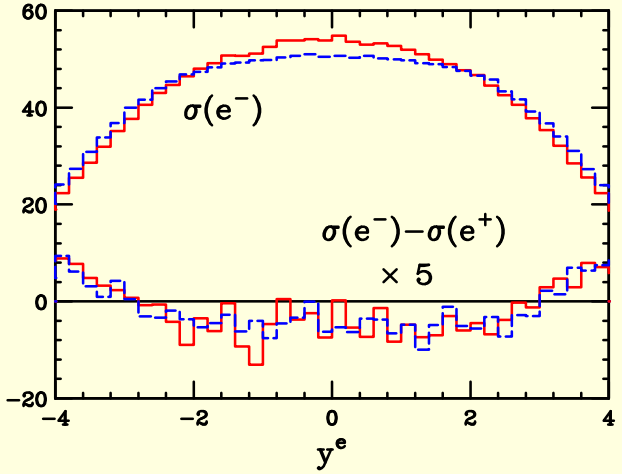
For $2 \to 1$ processes (W/Z and Higgs production), PYTHIA ME corrections are very similar to POWHEG; it implements the formula

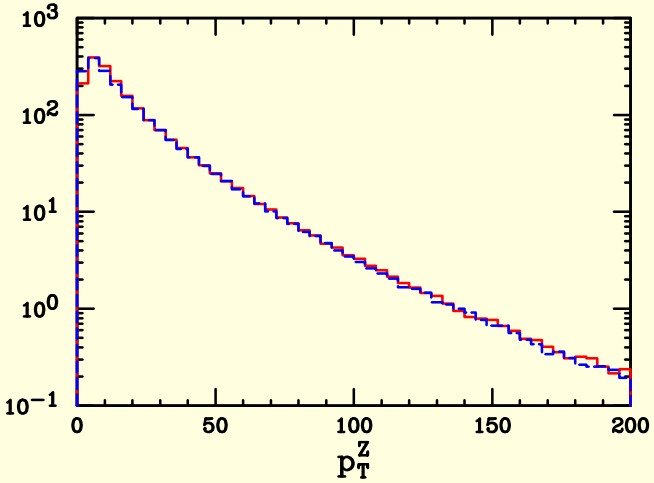
$$d\sigma = d\Phi_B \underbrace{B(\Phi_B)}_{\bar{B} \text{ in POWHEG}} \left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r \right], \qquad \Delta_t = \exp\left[- \int \theta(t_r - t) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right]$$

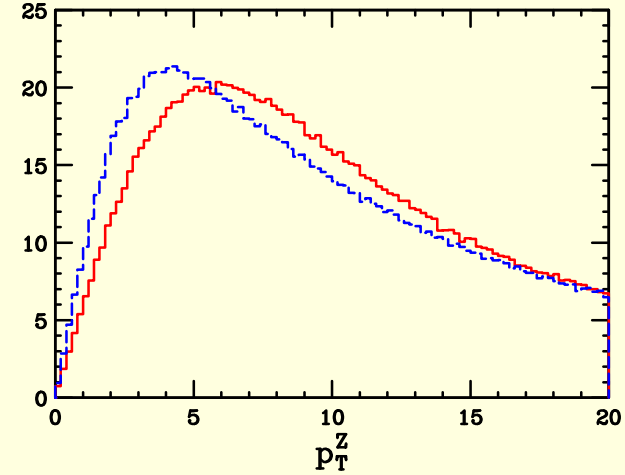
Dashes: PYTHIA X 1.172, Solid: POWHEG



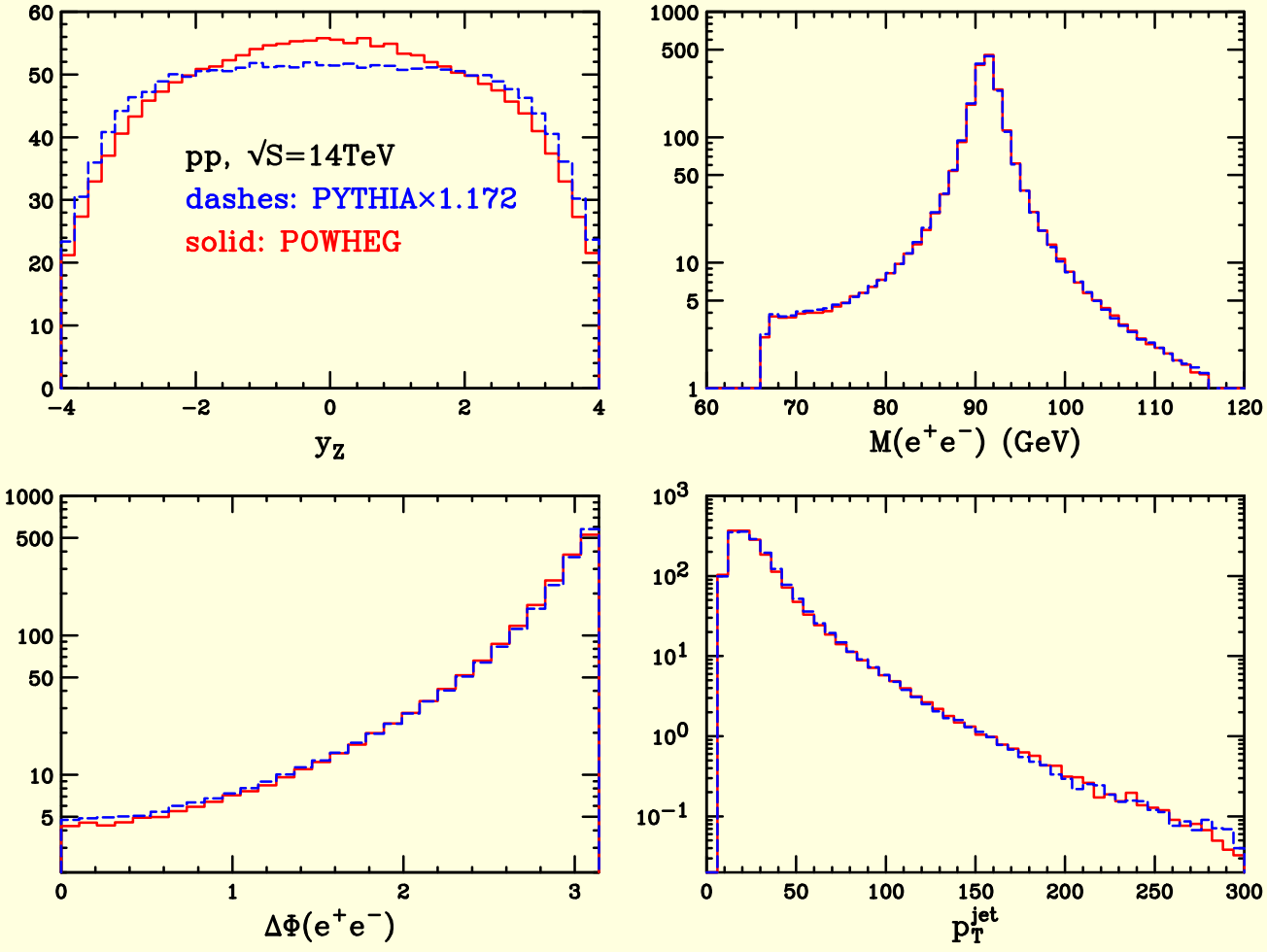

pp, √S=14 TeV
dashes: PYTHIA×1.172
solid: POWHEG
picobarn
per bin
p_T^e


$\sigma(e^-)$
$\sigma(e^-)-\sigma(e^+)$
× 5
y^e


p_T^Z

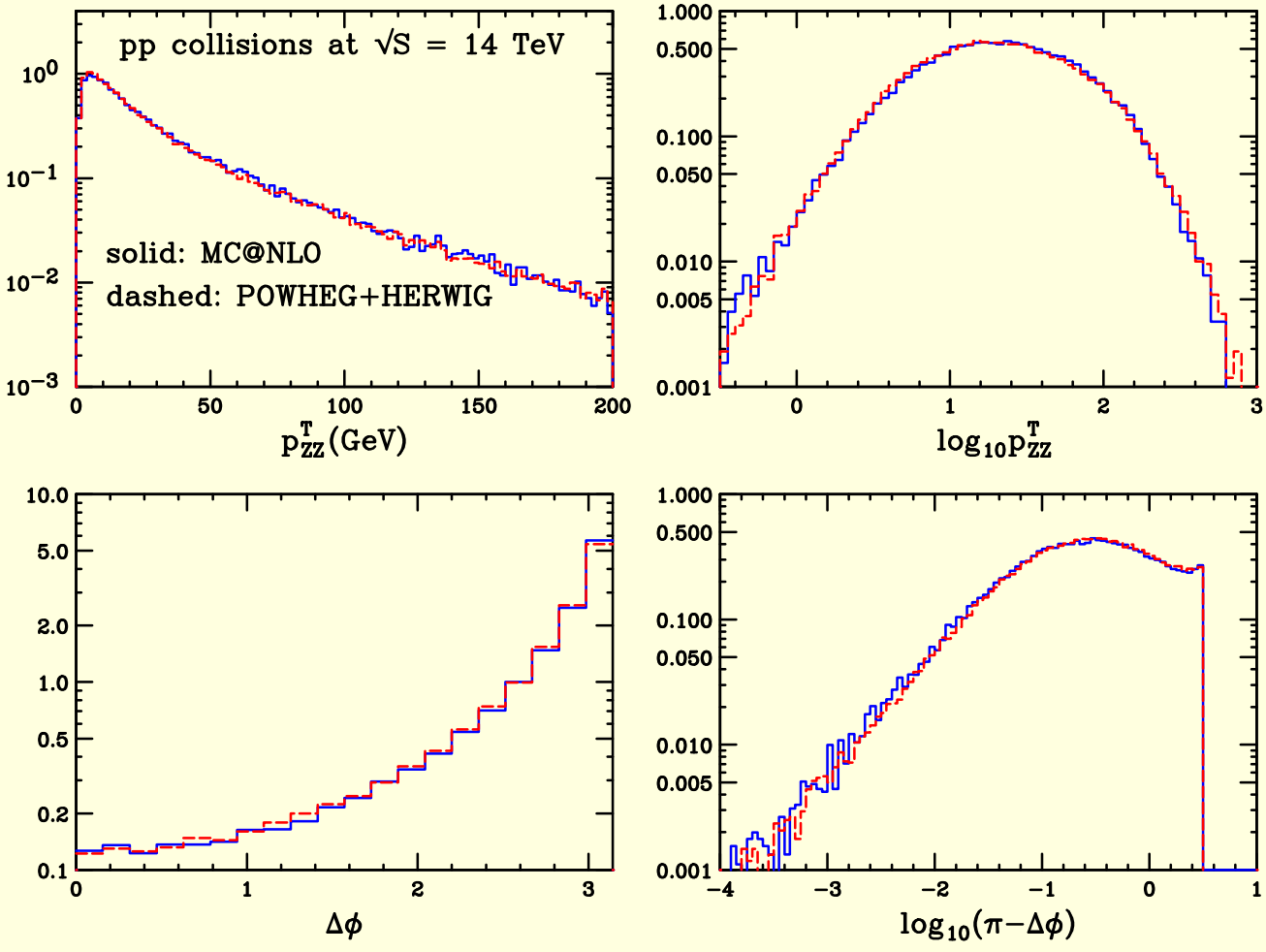

p_T^Z

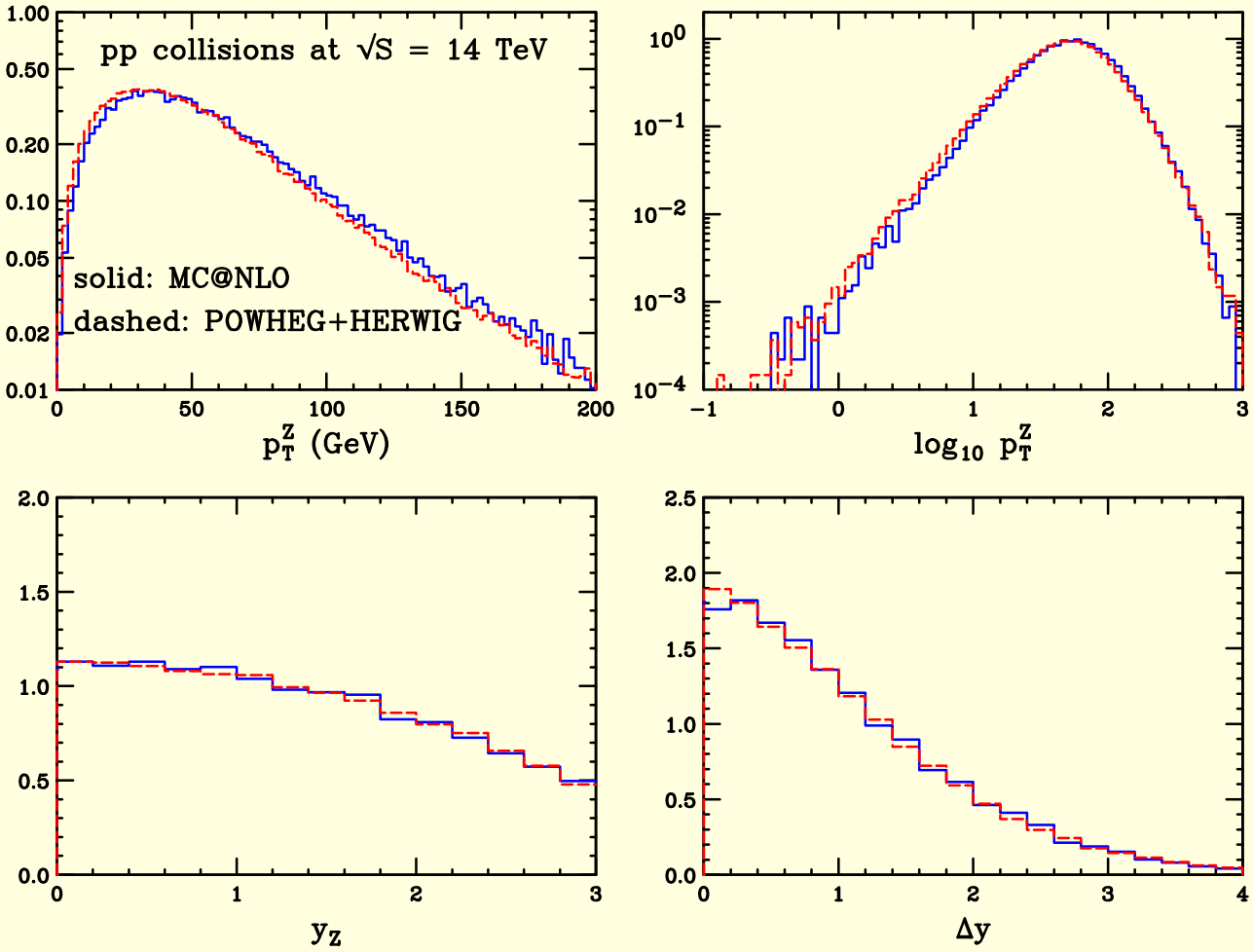
Different shape in y_Z distribution understood as NLO effect

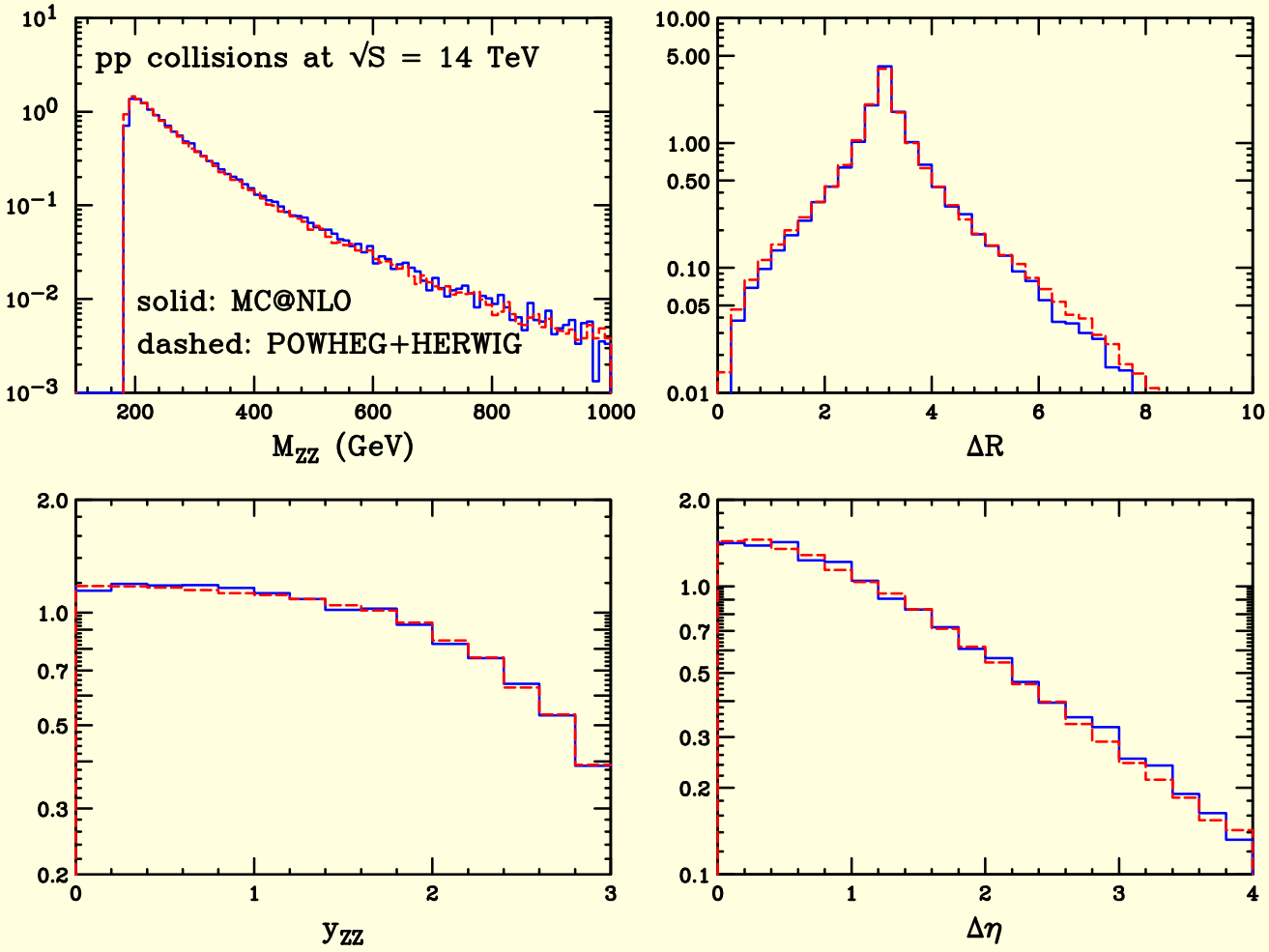


Comparisons of POWHEG+HERWIG vs. MC@NLO

Z pair production

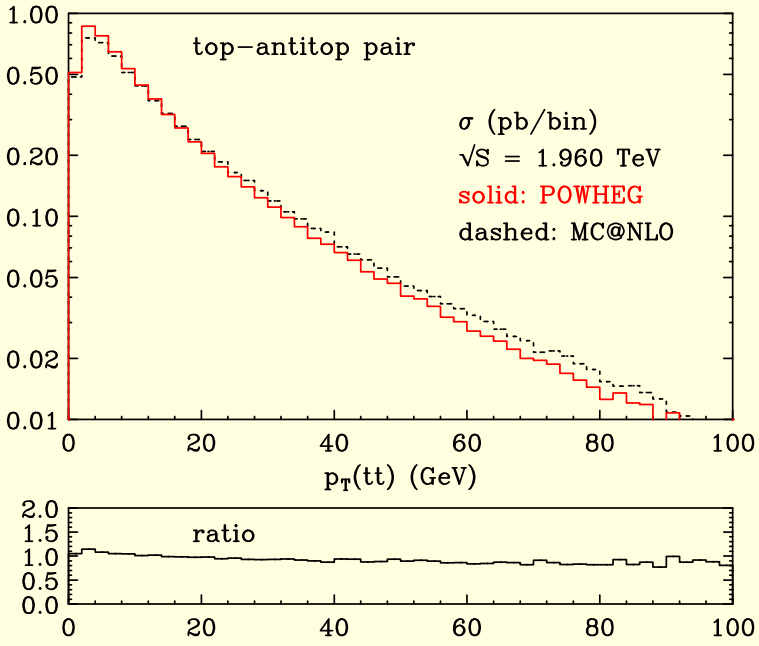


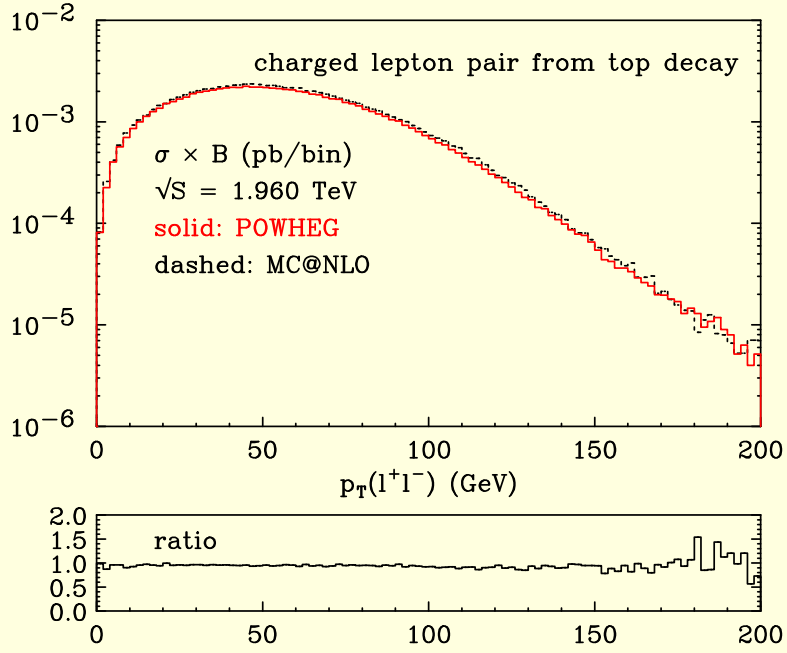

pp collisions at √S = 14 TeV
solid: MC@NLO
dashed: POWHEG+HERWIG
p_T^Z (GeV)
$\log_{10} p_T^Z$
y_Z
Δy



Remarkable agreement for most quantities;

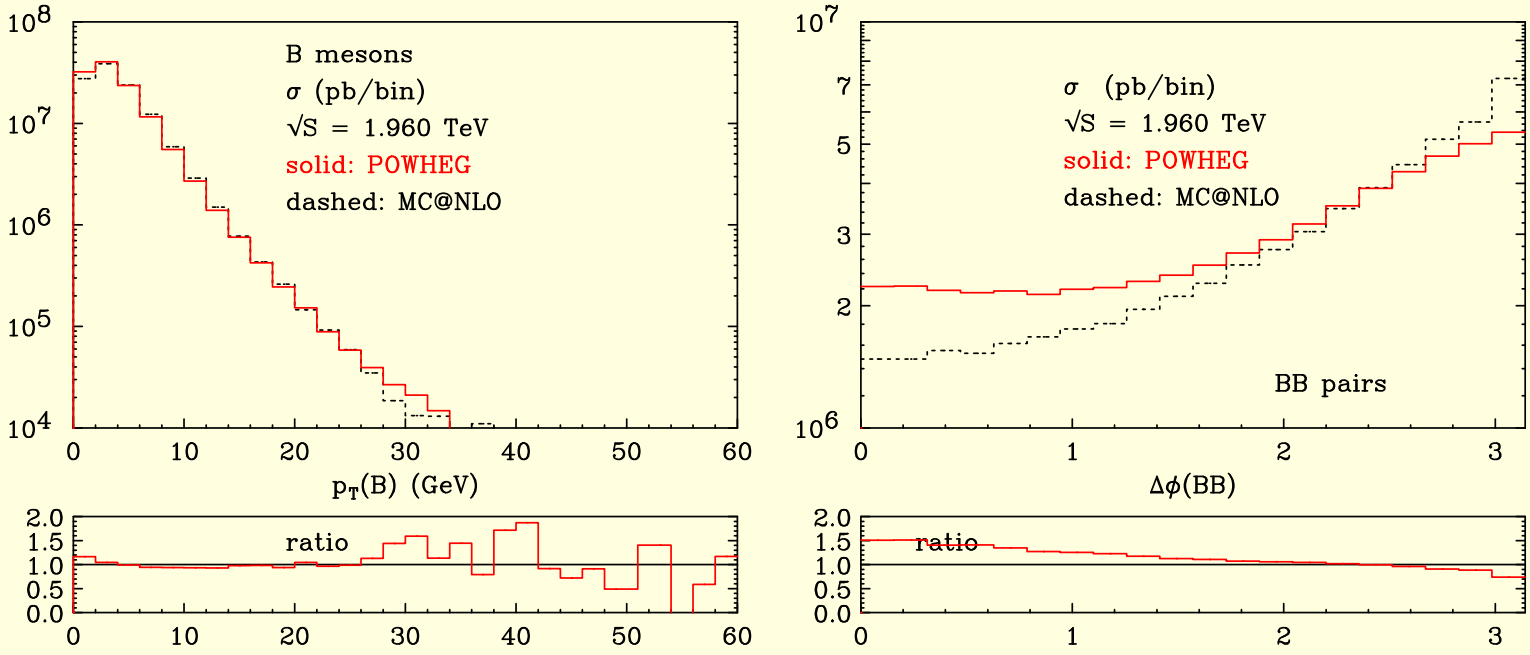
POWHEG and MC@NLO comparison: Top pair production





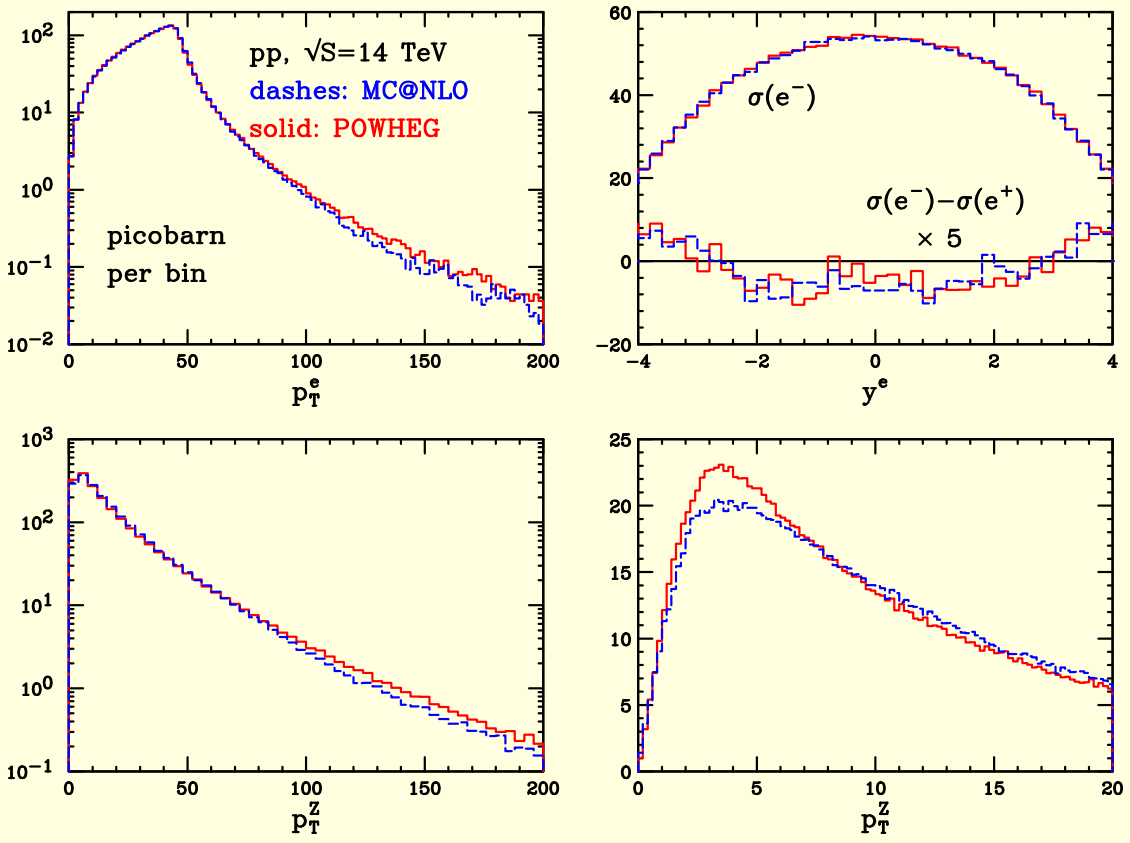
Good agreement for all observable considered
(differences can be ascribed to different treatment of higher order terms)

Bottom pair production



- Very good agreement For large scales (ZZ, $t\bar{t}$ production)
- Differences at small scales ($b\bar{b}$ at the Tevatron)
- POWHEG more reliable in extreme cases like $b\bar{b}, c\bar{c}$ at LHC
  (yields positive results, MC@NLO has problems with negative weights)

Z production: POWHEG+HERWIG vs. MC@NLO



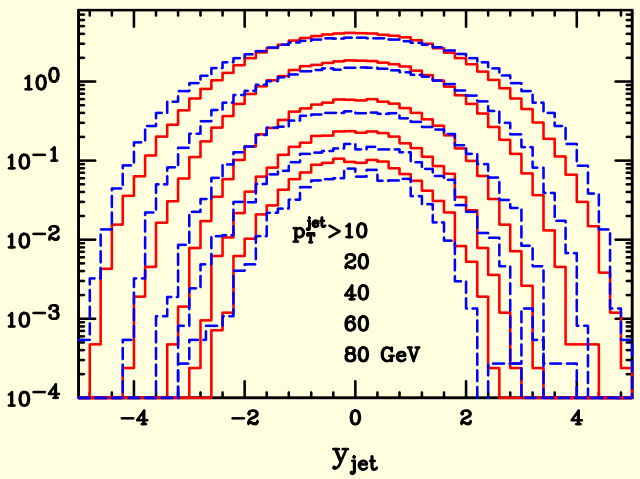
Small differences in high and low p_T region

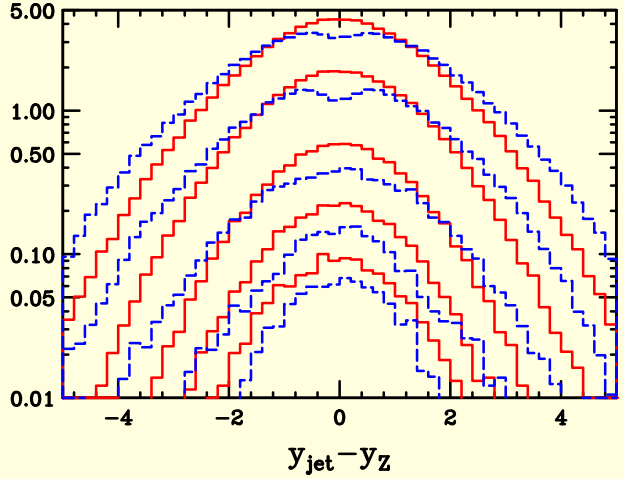
In some instances we do find differences!

Theese have been studied and understood in great details.

They can be attributed to subleading NNLO effects (i.e. beyond the declared accuracy of the methods.)

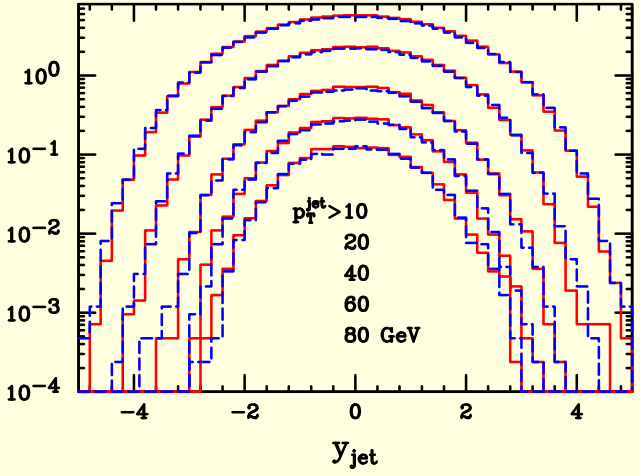
Z production: rapidity of hardest jet (TEVATRON)

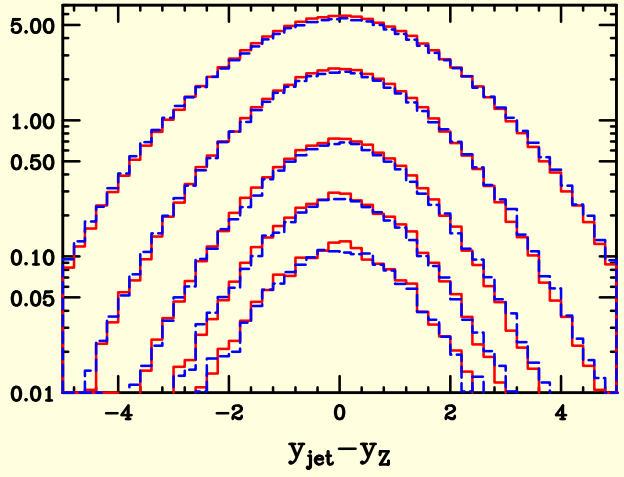




POWHEG+HERWIG

MC@NLO

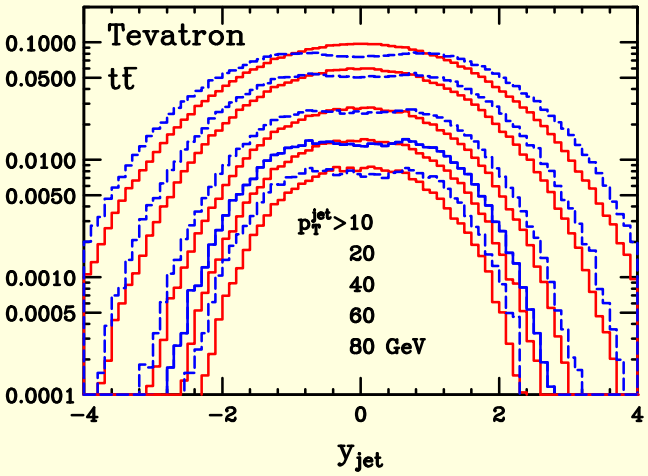


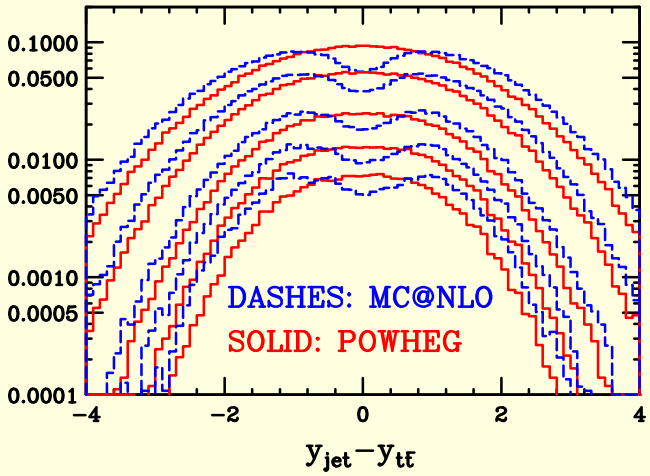


POWHEG+PYTHIA

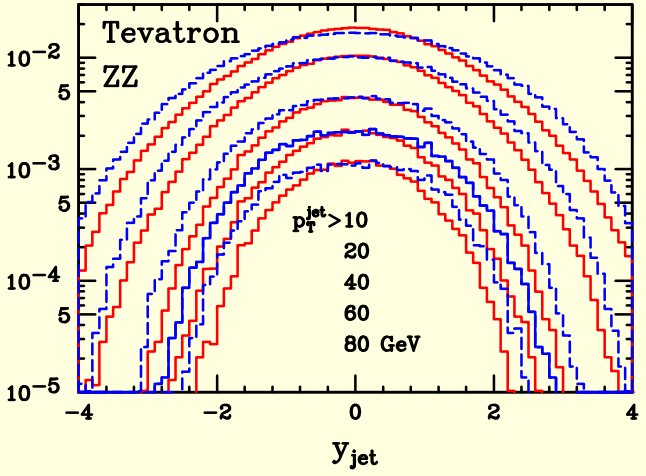
PYTHIA

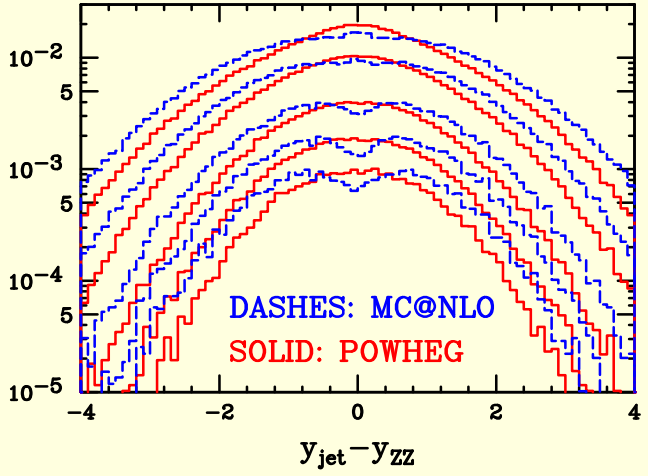
Dip in central region in MC@NLO also in $t\bar{t}$ and ZZ





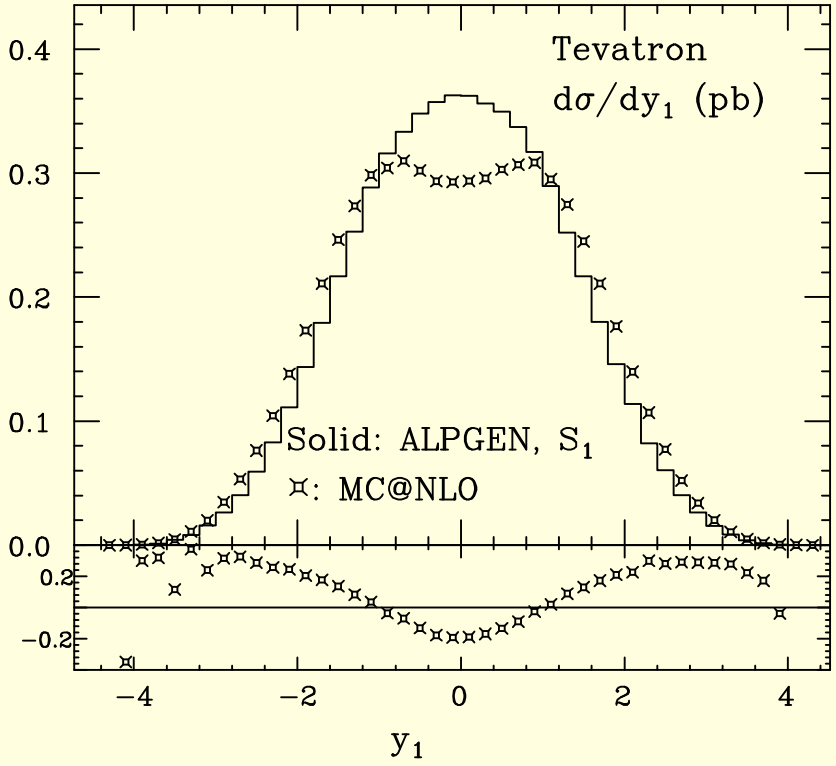
POWHEG+HERWIG
MC@NLO

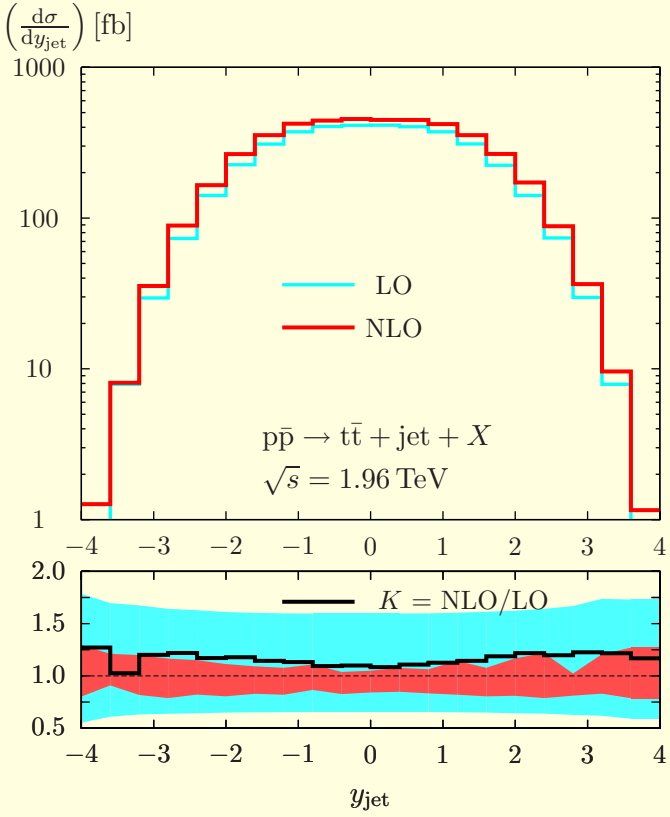




POWHEG+HERWIG
MC@NLO

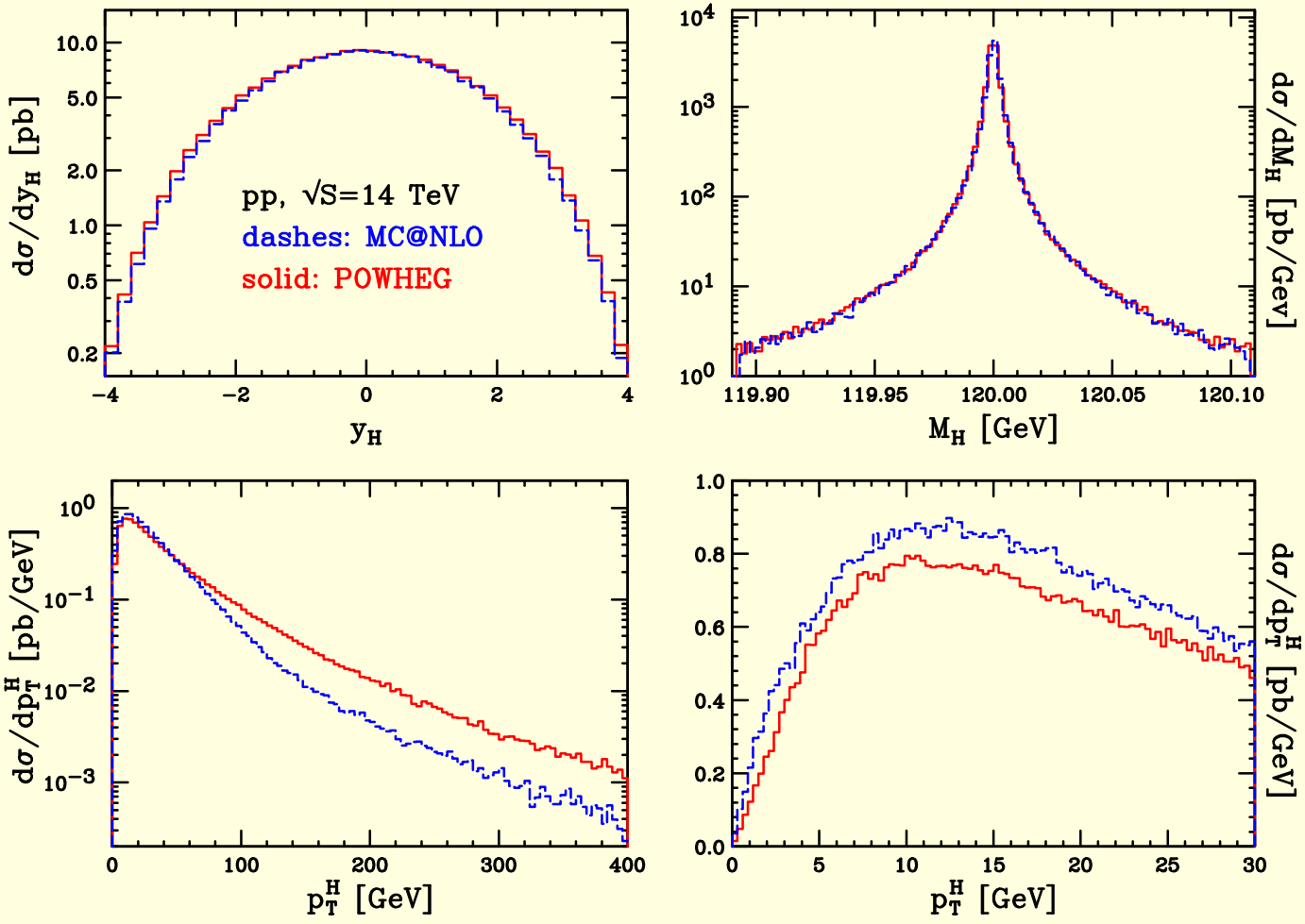
ALPGEN and $t\bar{t}+\text{jet}$ at NLO vs. MC@NLO



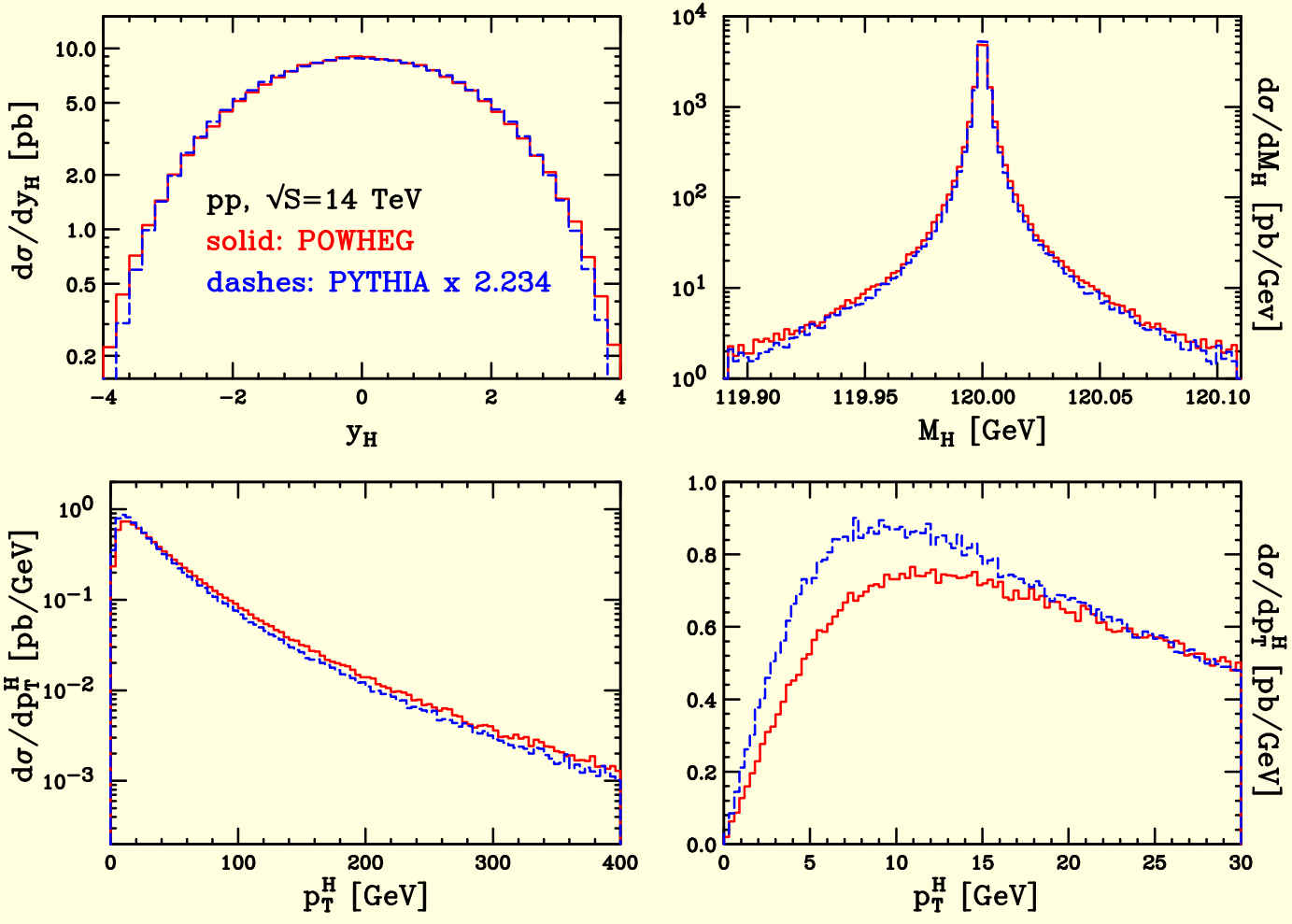


POWHEG distribution as in ALPGEN (Mangano,Moretti,Piccinini,Treccani,Nov.06) and in $t\bar{t}+\text{jet}$ at NLO (Dittmaier, Uwer, Weinzierl) : no dip present.

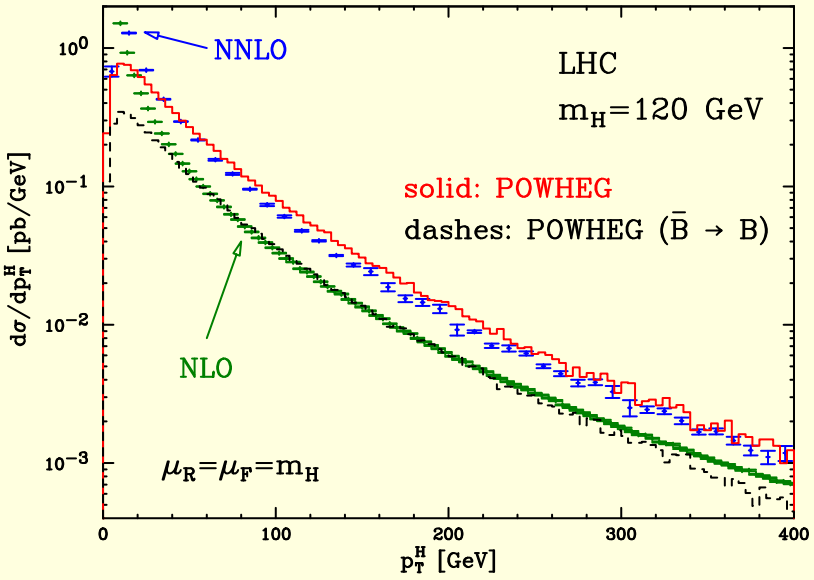
Higgs boson via gluon fusion at LHC

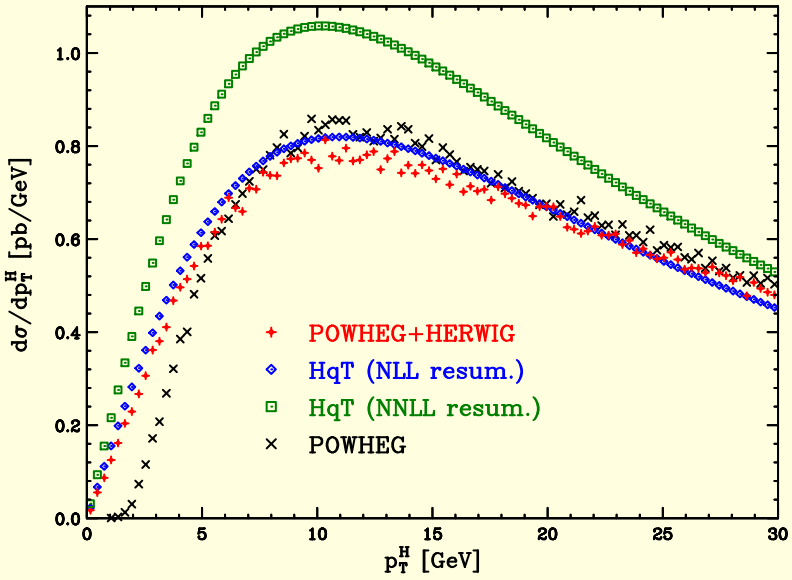


Higgs boson via gluon fusion at LHC



POWHEG vs. NNLO vs. NNLL



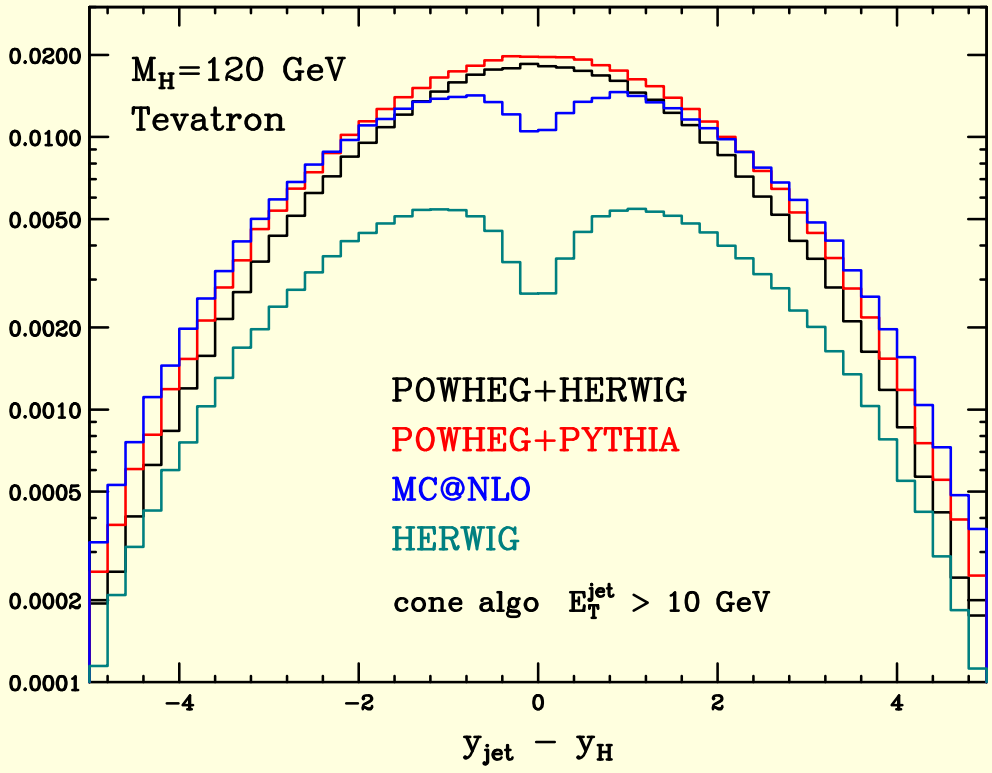


$$d\sigma = \bar{B}(\Phi_B)\, d\Phi_B \left\{ \Delta(\Phi_B, p_T^{\min}) + \Delta(\Phi_B, p_T) \frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r \right\}$$

$$\approx \frac{\bar{B}(\Phi_B)}{B(\Phi_B)} R(\Phi_B, \Phi_r)\, d\Phi_r = \{1 + \mathcal{O}(\alpha_s)\} R(\Phi)\, d\Phi$$

Better agreement with NNLO this way.

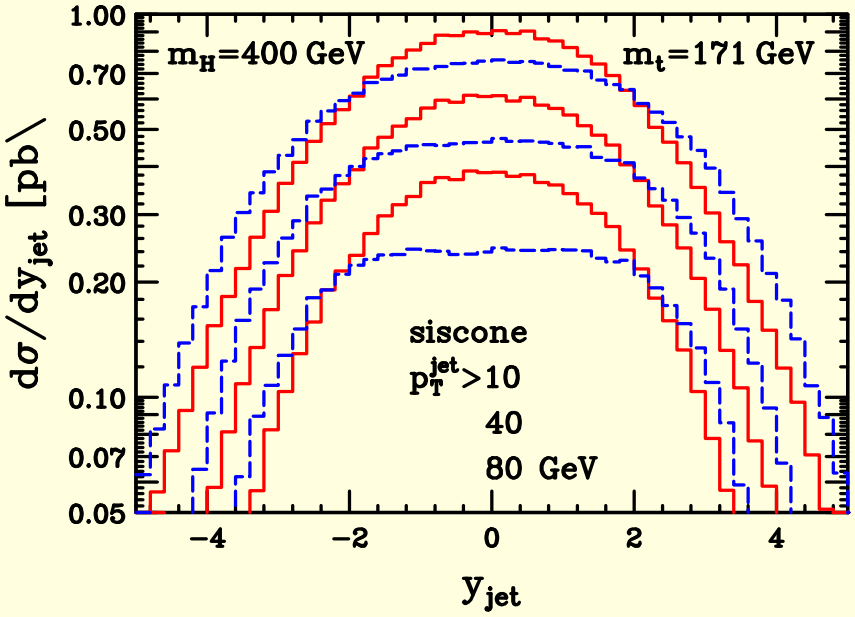
Jet rapidity in h production

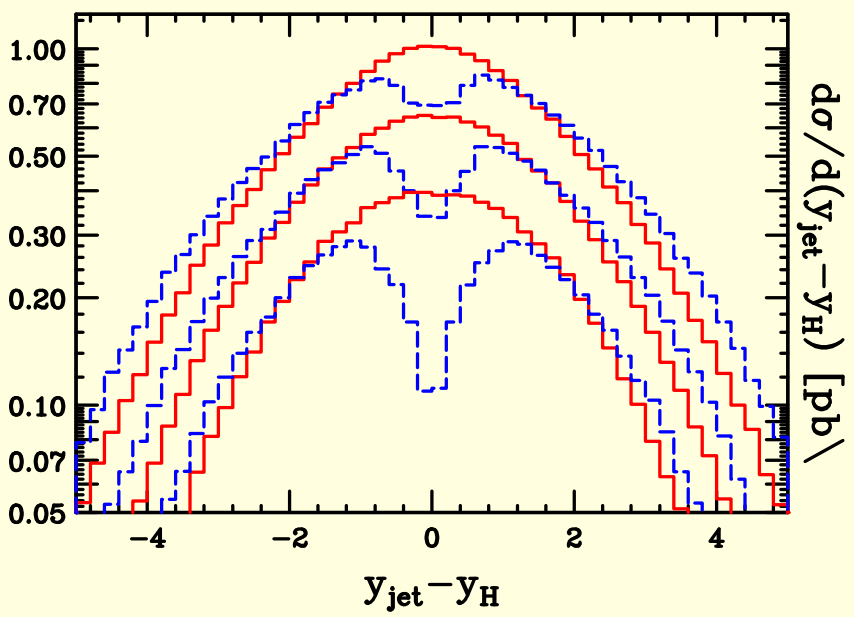


Dip in MC@NLO inerithed from even deeper dip in HERWIG

(MC@NLO tries to fill dead regions in HERWIG, a mismatch remains).

Gets worse for larger E_T cuts:





Questions:

Why MC@NLO has a dip in the hardest jet rapidity?

Why POWHEG has no dip? Is that because of the hardest p_T spectrum?

Hard p_T spectrum in POWHEG

We understand the cause; we keep it because yields results closer to NNLO;
There is enough flexibility to get rid of it, if one wants!
Go back to the POWHEG cross section:

$$d\sigma = \bar{B}(\Phi_B)\left[\Delta_{t_0} + \Delta_t \frac{R(\Phi)}{B(\Phi_B)} d\Phi_r\right], \qquad \Delta_t = \exp\left[-\int \theta(t_r - t)\frac{R(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r\right]$$

Break $R = R_s + R_f$, with R_f finite in collinear and soft limit, define

$$d\sigma' = \bar{B}^s(\Phi_B)\left[\Delta^s_{t_0} + \Delta^s_t \frac{R_s(\Phi)}{B(\Phi_B)} d\Phi_r\right] + R_f(\Phi) d\Phi$$

with:

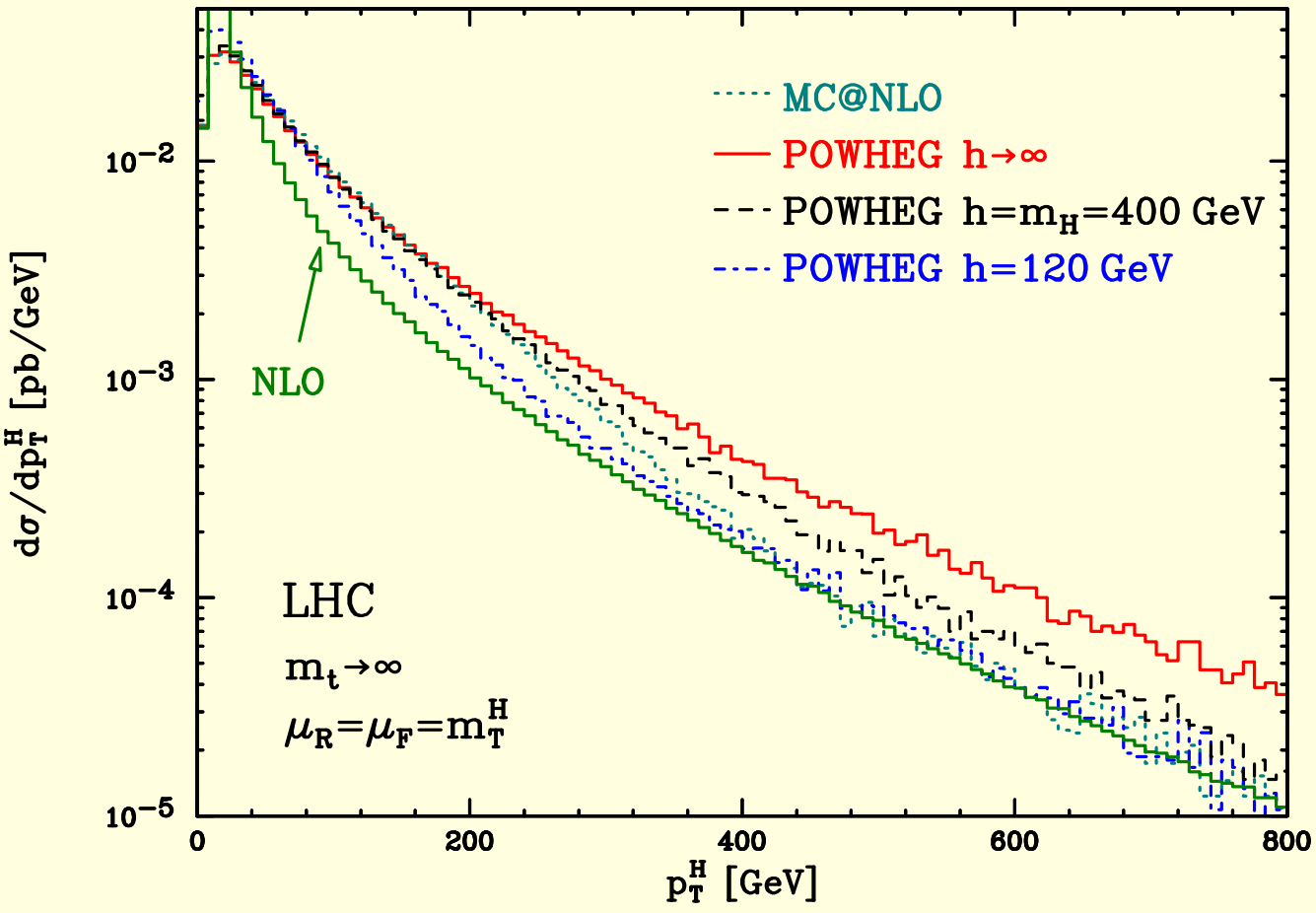
$$\Delta^s_t = \exp\left[-\int \theta(t_r - t)\frac{R^s(\Phi_B, \Phi_r)}{B(\Phi_B)} d\Phi_r\right].$$

Easy to prove that: $d\sigma'$ is equivalent to $d\sigma$.
In other words, the part of the real cross section that is treated with the Shower technique can be varied.

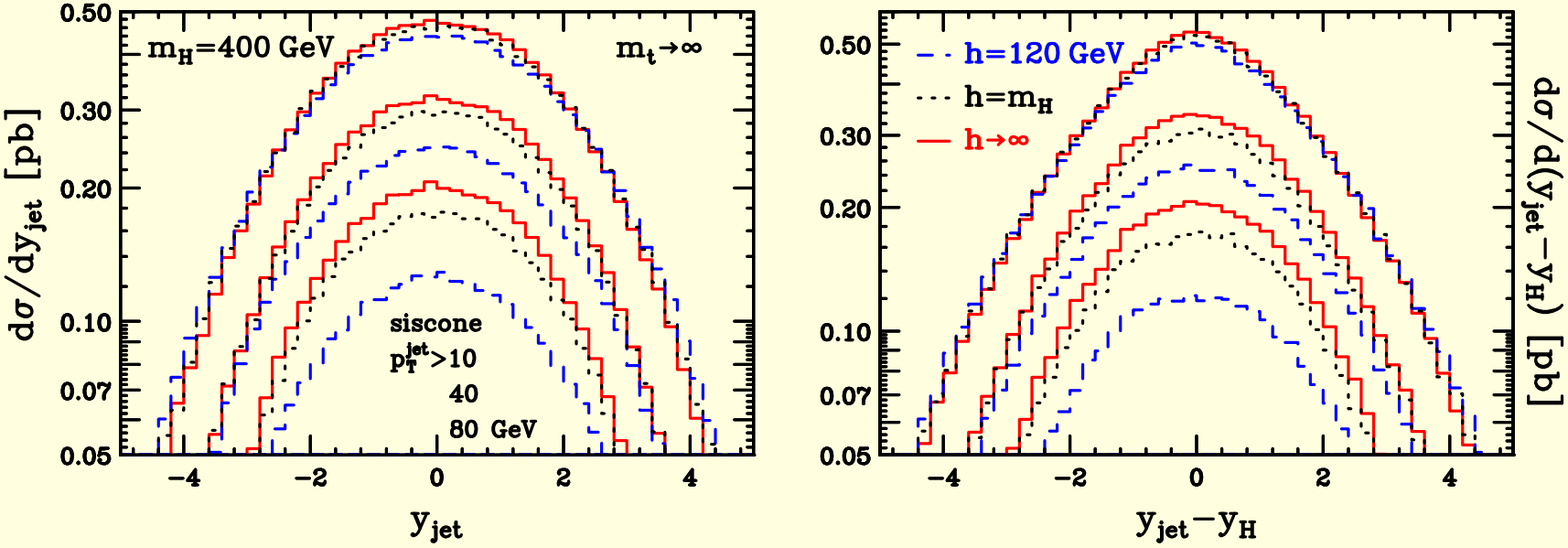
$$R_s = R\frac{h^2}{k_T^2 + h^2}$$

$$R_f = R\frac{k_T^2}{k_T^2 + h^2}$$

Agrees with NLO at high p_T.



No new features (dips and the like) arise in the other distributions:



So: high k_T cross section and dips are unrelated issues.

Why is there a dip in MC@NLO?

Write the MC@NLO hardest jet cross section in the POWHEG language;
Hardest emission (P.N., 2004) can be written as

$$d\sigma = \underbrace{\bar{B}^{\rm HW}(\Phi_B)d\Phi_B}_{S\,\text{event}}\left[\underbrace{\Delta^{\rm HW}_{t_0} + \Delta^{\rm HW}_t \frac{R^{\rm HW}(\Phi)}{B(\Phi_B)} d\Phi^{\rm HW}_r}_{\text{HERWIG shower}}\right] + \left[\underbrace{R(\Phi) - R^{\rm HW}(\Phi)}_{H\,\text{event}}\right] d\Phi$$

$$\bar{B}^{\rm HW}(\Phi_B) = B(\Phi_B) + \underbrace{\left[\underbrace{V(\Phi_B)}_{\text{infinite}} + \underbrace{\int R^{\rm HW}(\Phi_B,\Phi_r)\,d\Phi_r}_{\text{infinite}}\right]}_{\text{finite}}$$

(Imagine that soft and collinear singularities in $R^{\rm HW}$ are regulated as in V!).
Like POWHEG with $R_s = R^{\rm HW}$. But now $R_f = R - R^{\rm HW}$ can be negative.
This formula illustrates why MC@NLO and POWHEG are equivalent at NLO.
But differences can arise at NNLO ...

For large k_T:

$$d\sigma = \left[\frac{\bar{B}^{\rm HW}(\Phi_{_B})}{B(\Phi_{_B})} R^{\rm HW}(\Phi) + R(\Phi) - R^{\rm HW}(\Phi) \right] d\Phi_{_B}\, d\Phi_r^{\rm HW}$$

$$= \underbrace{R(\Phi) d\Phi}_{\text{no dip}} + \underbrace{\left(\frac{\bar{B}^{\rm HW}(\Phi_{_B})}{B(\Phi_{_B})} - 1 \right)}_{\mathcal{O}(\alpha_s),\ \text{but large for Higgs}} \underbrace{R^{\rm HW}(\Phi)}_{\text{Pure Herwig dip}} \; d\Phi$$

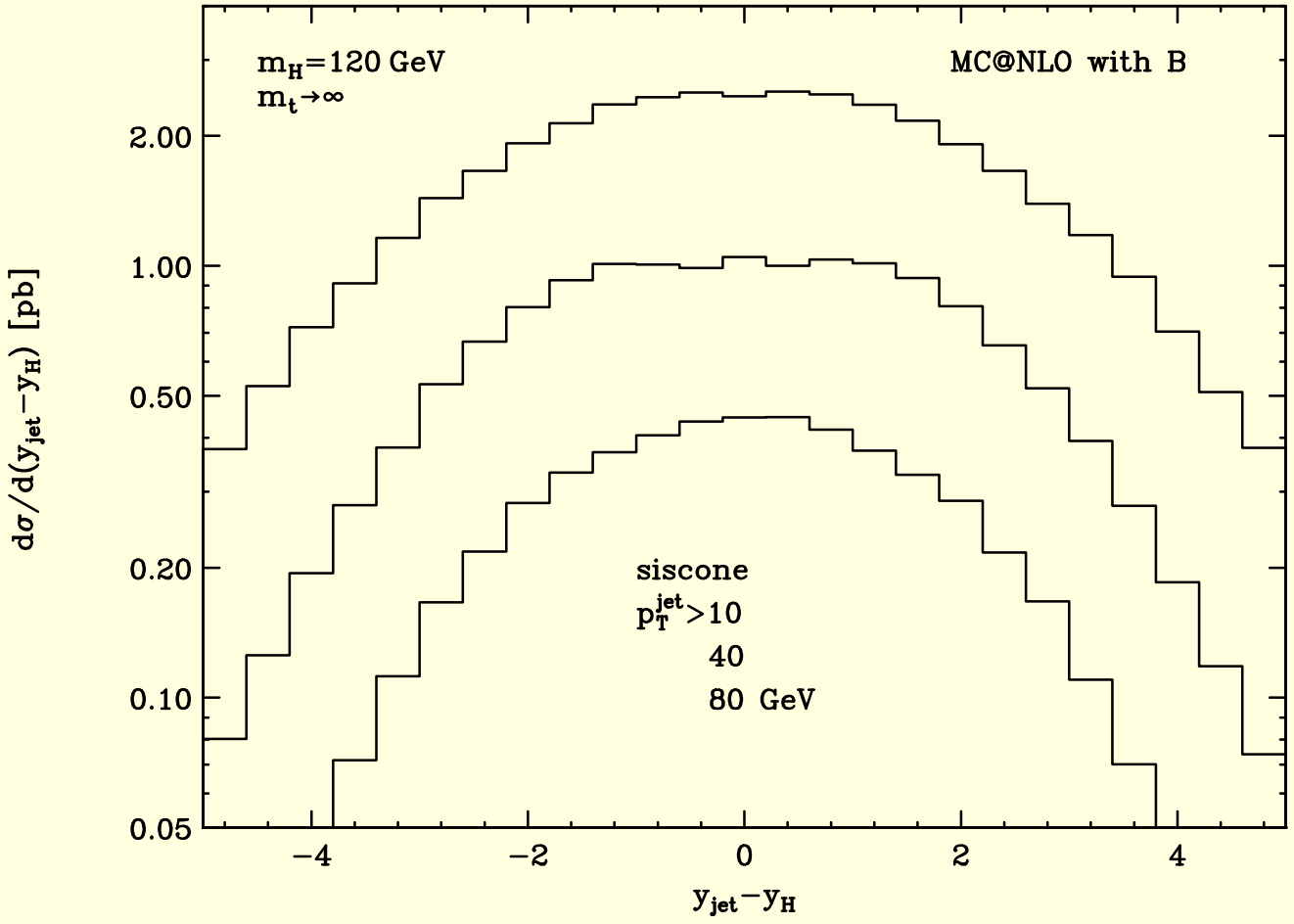
So: a contribution with a dip is added to the exact NLO result;

The contribution is $\mathcal{O}(\alpha_s R)$, i.e. NNLO!

Can we test this hypothesis? Replace $\bar{B}^{\rm HW}(\Phi_n) \Rightarrow B(\Phi_n)$ in MC@NLO!

the dip should disappear ...

MC@NLO with B^{HW} replaced by B



No visible dip is present! (on the right track, more studies needed cd Does...)

Towards automation: the POWHEG BOX

The MIB (Milano-Bicocca) group (Alioli, Oleari, Re, P.N.) is working on an automatic implementation of POWHEG for generic NLO processes.

This framework is being tested in the process $h\,h \to Z + 1\mathrm{jet}$.

The POWHEG BOX

Build a computer code framework, such that, given the Born cross section, the finite part of the virtual corrections, and the real graph cross section, one builds immediately a POWHEG generator. More precisely, the user must supply:

- The Born phase space
- The lists of Born and Real processes (i.e. $u\,\bar{s} \to W^+ c\,\bar{c}$, etc.)
- The Born squared amplitudes $\mathcal{B} = |\mathcal{M}|^2$, $\mathcal{B}_{ij}$, $\mathcal{B}_{j,\mu_j,\mu'_j}$, for all relevant partonic processes; $\mathcal{B}_{ij}$ is the colour ordered Born amplitude squared, $\mathcal{B}_{j,\mu\nu}$ is the spin correlated amplitude, where j runs over all external gluons in the amplitude. All these amplitudes are common ingredient of an NLO calculation.
- The Real squared amplitude, for all relevant partonic processes. This may also be obtained by interfacing the program to MADGRAPH.
- The finite part of the virtual amplitude contribution, for all relevant partonic processes.

Strategy

Use the FKS framework according to the general formulation of POWHEG given in (Frixione, Oleari, P.N. 2007), hiding all FKS implementation details.
In other words, we use FKS, but the user needs not to understand it.
(Attempts to use the popular Catani-Seymour method
have turned out to be too cumbersome).
It includes:

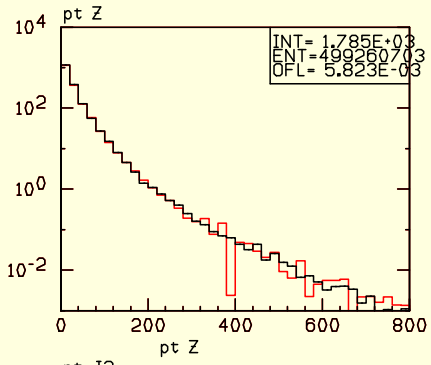
- The phase space for ISR and FSR, according to FNO2006.
- The combinatorics, the calculation of all R_α, the soft and coll. limits
- The calculation of $\tilde{B}$
- The calculation of the upper bounds for the generation of radiation
- The generation of radiation
- Writing the event to the Les Houches interface

It works! Lots of testing needed now ...
Byproduct: generic NLO implementation using the FKS method

Case study: $Z + \mathrm{jet}$ production

Get virtual matrix elements from `MCFM`;
Compare first NLO predictions obtained with `MCFM` and the `POWHEG BOX`

Virtual corrections are the same, but subtraction terms, soft and collinear remnants are all different; non trivial test of setup;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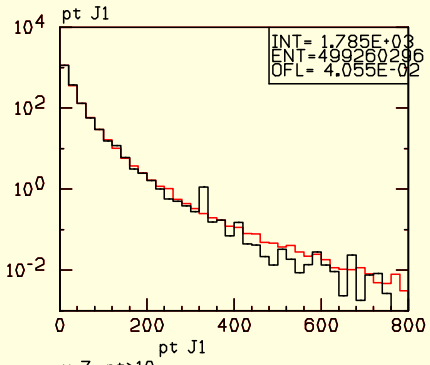
ENT=4992607O3
OFL= 5.823E-03
pt Z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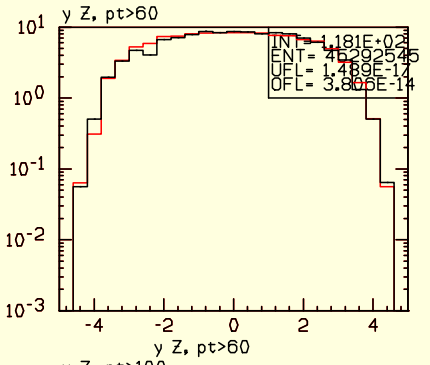
pt J1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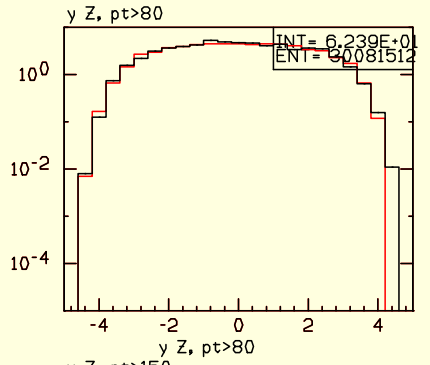
OFL= 3.866E-14
y Z, pt>60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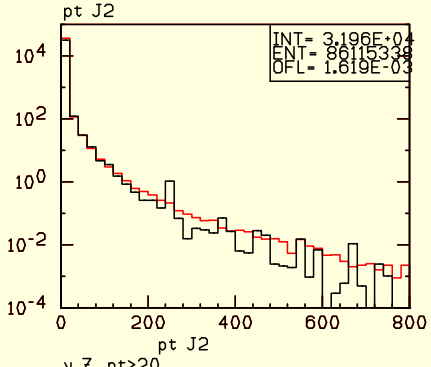
ENT= 3008151?
y Z, pt>80


pt J2
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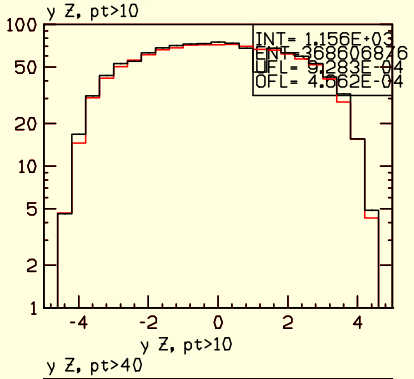
ENT= 86115338
OFL= 1.619E-03
pt J2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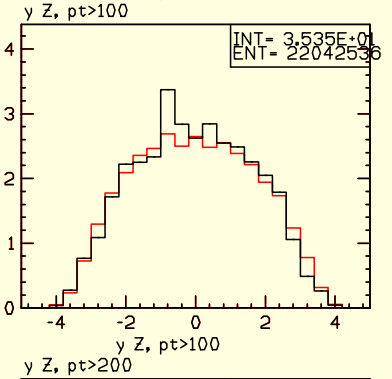
OFL= 4.662E-04
y Z, pt>10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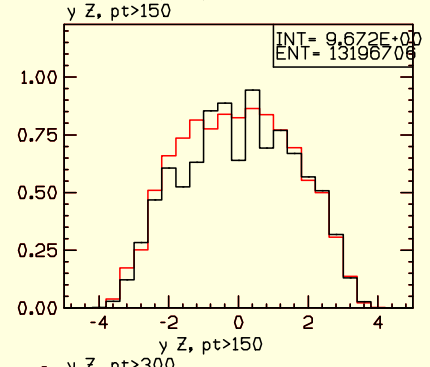
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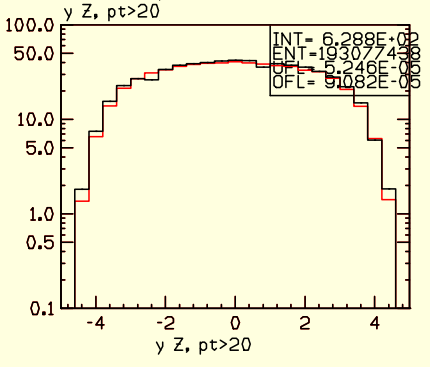
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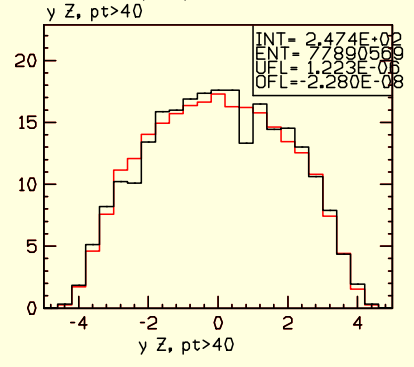
y Z, pt>20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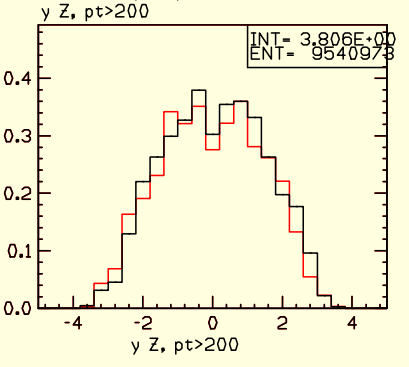
OFL=-2.280E-08
y Z, pt>40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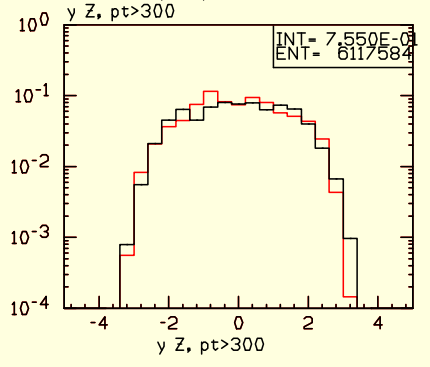
y Z, pt>200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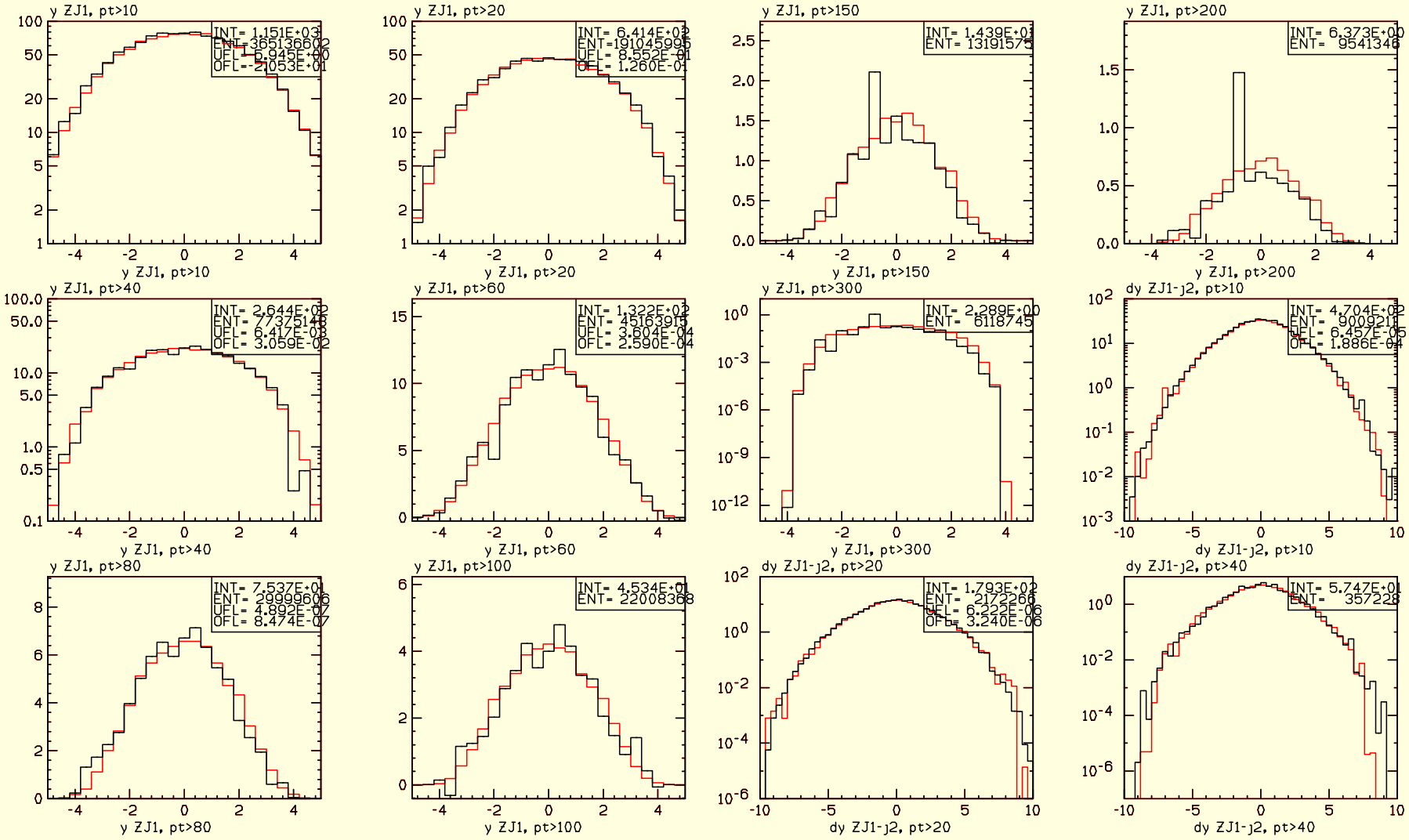
ENT= 6117584
y Z, pt>300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UFL= 6.945E+00
OFL= 2.053E+01
y ZJ1, pt>10
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INT= 6.414E+02
ENT=191045995
UFL= 8.552E-01
OFL= 1.260E-01
y ZJ1, pt>20
y ZJ1, pt>150
INT= 1.439E+01
ENT= 13191575
y ZJ1, pt>150
y ZJ1, pt>200
INT= 6.373E+00
ENT= 9541346
y ZJ1, pt>200
y ZJ1, pt>40
INT= 2.644E+02
ENT= 77375148
UFL= 6.417E-03
OFL= 3.059E-02
y ZJ1, pt>40
y ZJ1, pt>60
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ENT= 45163915
UFL= 3.604E-04
OFL= 2.590E-04
y ZJ1, pt>60
y ZJ1, pt>300
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y ZJ1, pt>300
dy ZJ1-j2, pt>10
INT= 4.704E+02
ENT= 9009211
UFL= 6.457E-05
OFL= 1.886E-04
dy ZJ1-j2, pt>10
y ZJ1, pt>80
INT= 7.537E+01
ENT= 29999606
UFL= 4.892E-07
OFL= 8.474E-07
y ZJ1, pt>80
y ZJ1, pt>100
INT= 4.534E+01
ENT= 22008368
y ZJ1, pt>100
dy ZJ1-j2, pt>20
INT= 1.793E+02
ENT= 2172266
UFL= 6.222E-06
OFL= 3.240E-06
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INT= 5.747E+01
ENT= 357228
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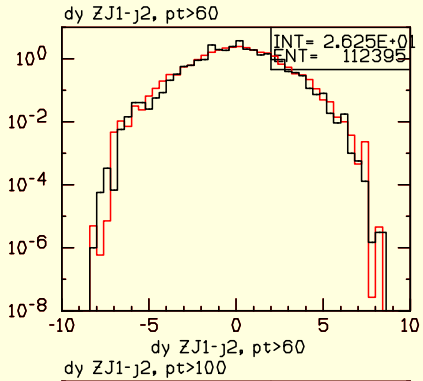
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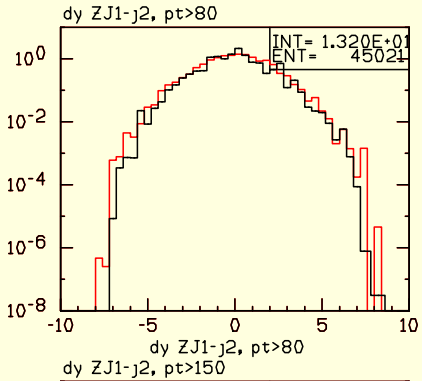
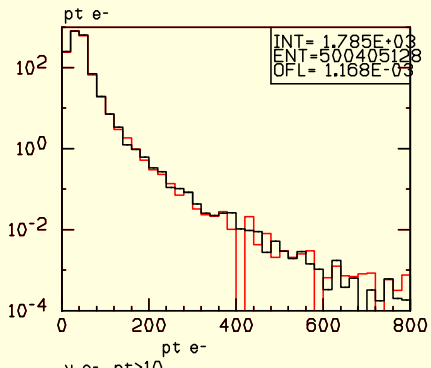
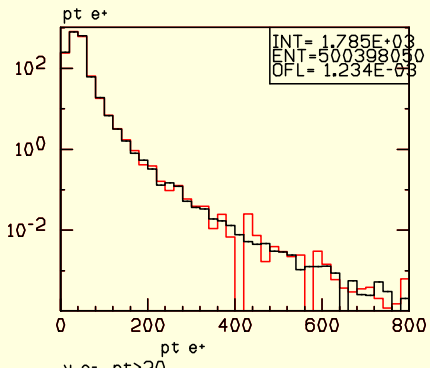
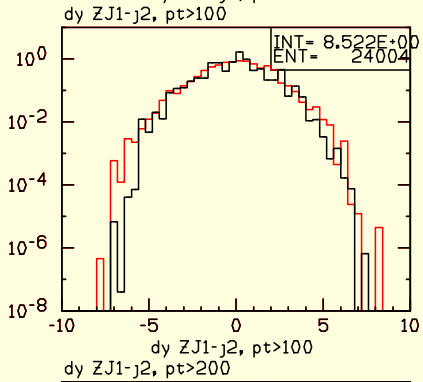
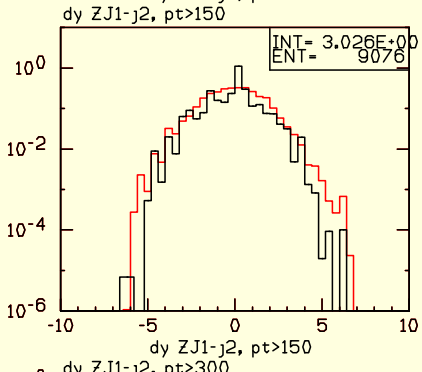
ENT= 112395
dy ZJ1-j2, pt>60

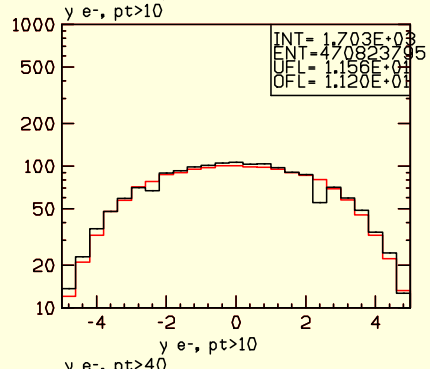

dy ZJ1-j2, pt>80
INT= 1.320E+01
ENT= 45021
dy ZJ1-j2, pt>80

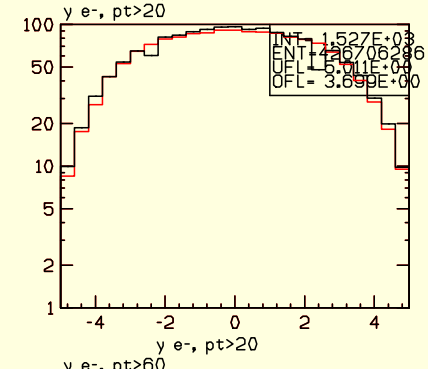

pt e-
INT= 1.785E+03
ENT=500405128
OFL= 1.168E-03
pt e-

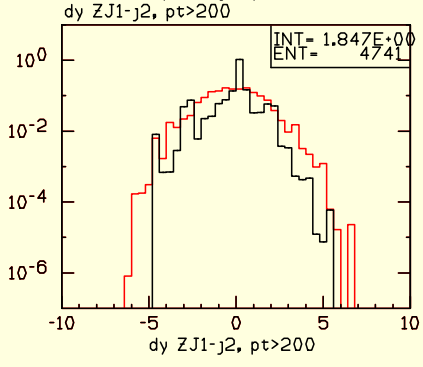

pt e+
INT= 1.785E+03
ENT=500398050
OFL= 1.234E-03
pt e+

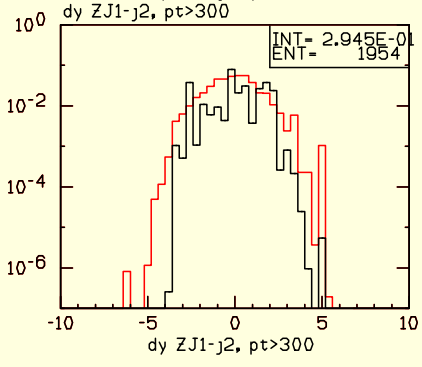

dy ZJ1-j2, pt>100
INT= 8.522E+00
ENT= 24004
dy ZJ1-j2, pt>100

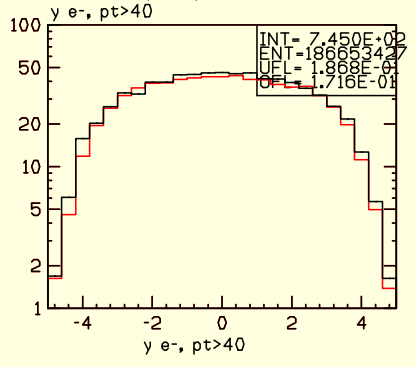

dy ZJ1-j2, pt>150
INT= 3.026E+00
ENT= 9076
dy ZJ1-j2, pt>150

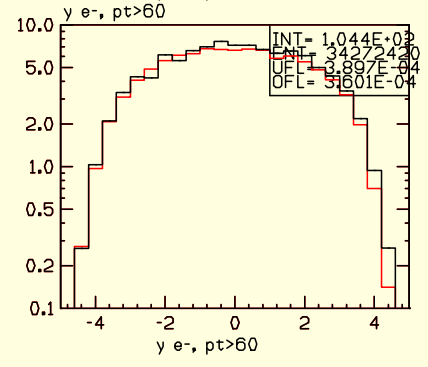

y e-, pt>10
INT= 1.703E+03
ENT=470823795
UFL= 1.156E-01
OFL= 1.120E-01
y e-, pt>10

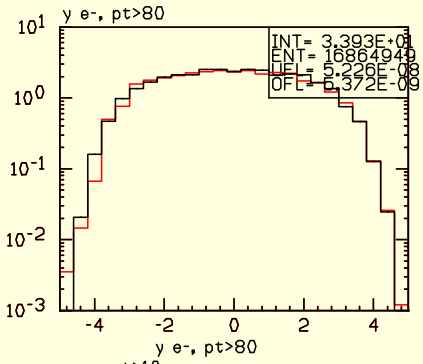

y e-, pt>20
INT= 1.527E+03
ENT=426706286
UFL= 5.011E-04
OFL= 3.699E+00
y e-, pt>20

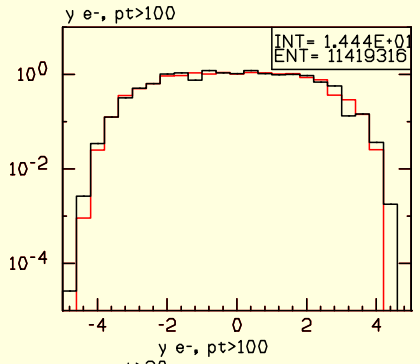

dy ZJ1-j2, pt>200
INT= 1.847E+00
ENT= 4741
dy ZJ1-j2, pt>200

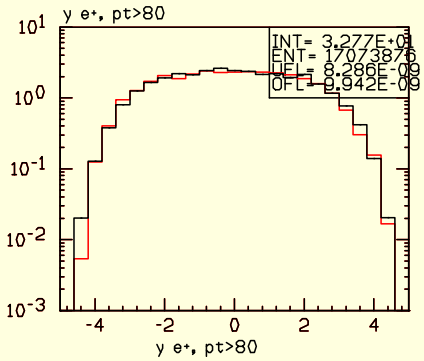

dy ZJ1-j2, pt>300
INT= 2.945E-01
ENT= 1954
dy ZJ1-j2, pt>300

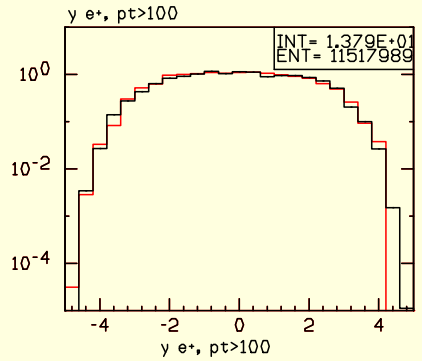

y e-, pt>40
INT= 7.450E+02
ENT=186653427
UFL= 1.868E-01
OFL= 1.716E-01
y e-, pt>40

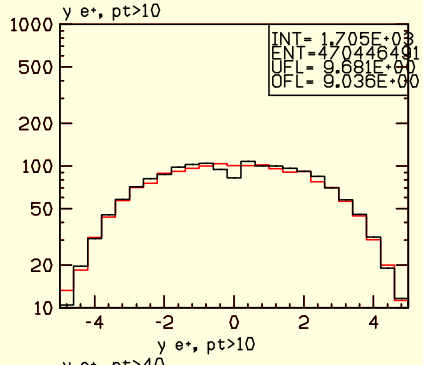

y e-, pt>60
INT= 1.044E+03
ENT= 342724 0
UFL= 3.897E-04
OFL= 5.601E-04
y e-, pt>60

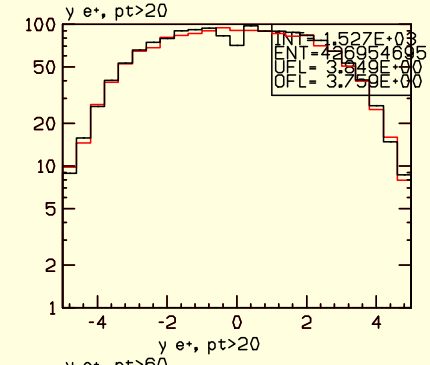

y e-, pt>80
INT= 3.393E+03
ENT= 16864949
UFL= 5.226E-08
OFL= 6.372E-09
y e-, pt>80

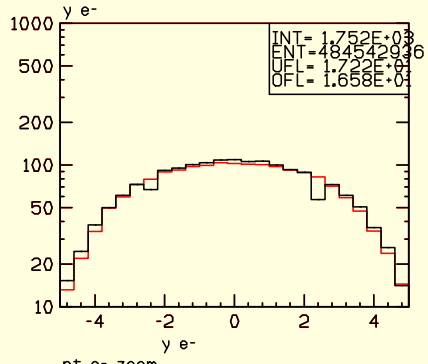

y e-, pt>100
INT= 1.444E+01
ENT= 11419316
y e-, pt>100

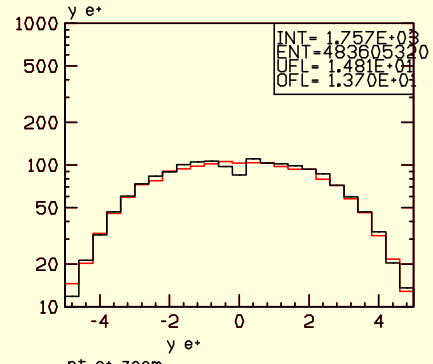

y e+, pt>80
INT= 3.277E+03
ENT= 17073876
UFL= 8.286E-09
OFL= 9.942E-09
y e+, pt>80

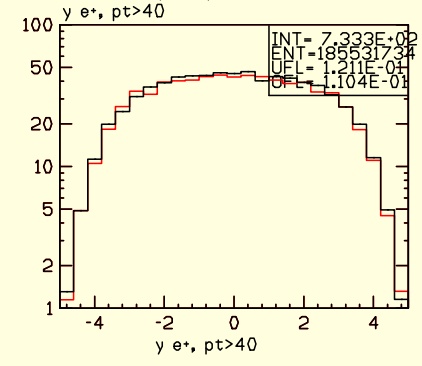

y e+, pt>100
INT= 1.379E+01
ENT= 11517989
y e+, pt>100

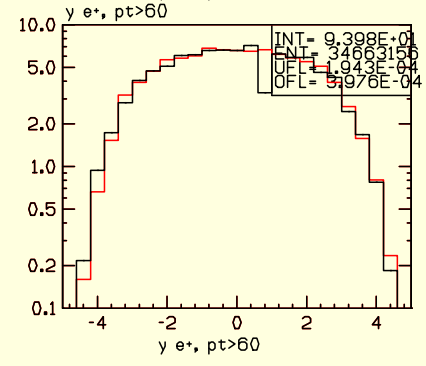

y e+, pt>10
INT= 1.705E+03
ENT=470446491
UFL= 9.681E-07
OFL= 9.036E+00
y e+, pt>10

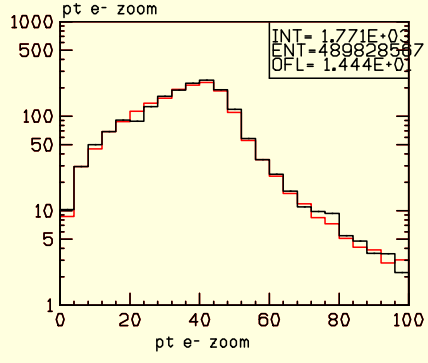

y e+, pt>20
INT= 1.527E+03
ENT=426954695
UFL= 3.549E-06
OFL= 3.759E+00
y e+, pt>20

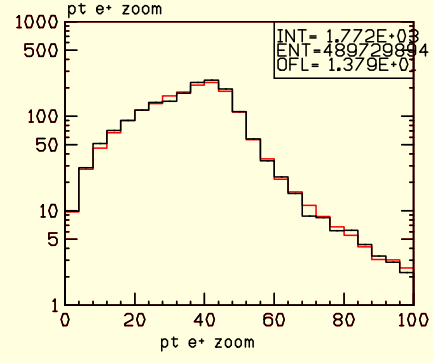

y e-
INT= 1.752E+03
ENT=484542936
UFL= 1.722E-07
OFL= 1.658E+01
y e-

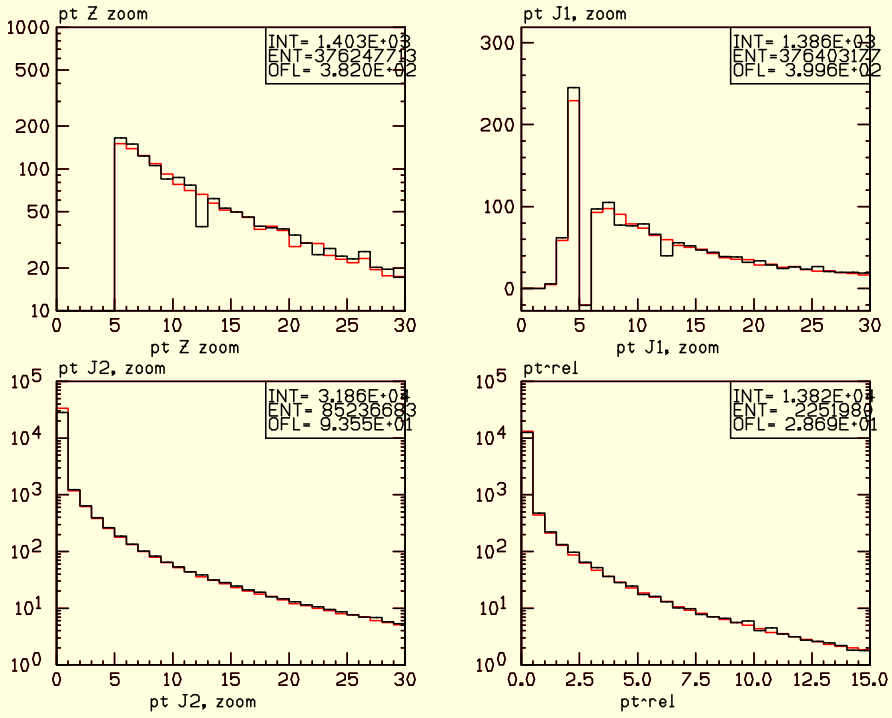

y e+
INT= 1.757E+03
ENT=483605320
UFL= 1.481E-07
OFL= 1.370E+01
y e+


y e+, pt>40
INT= 7.333E+02
ENT=185531734
UFL= 1.211E-04
OFL= 1.104E-01
y e+, pt>40


y e+, pt>60
INT= 9.398E+01
ENT= 34663156
UFL= 1.943E-04
OFL= 9.976E-04
y e+, pt>60

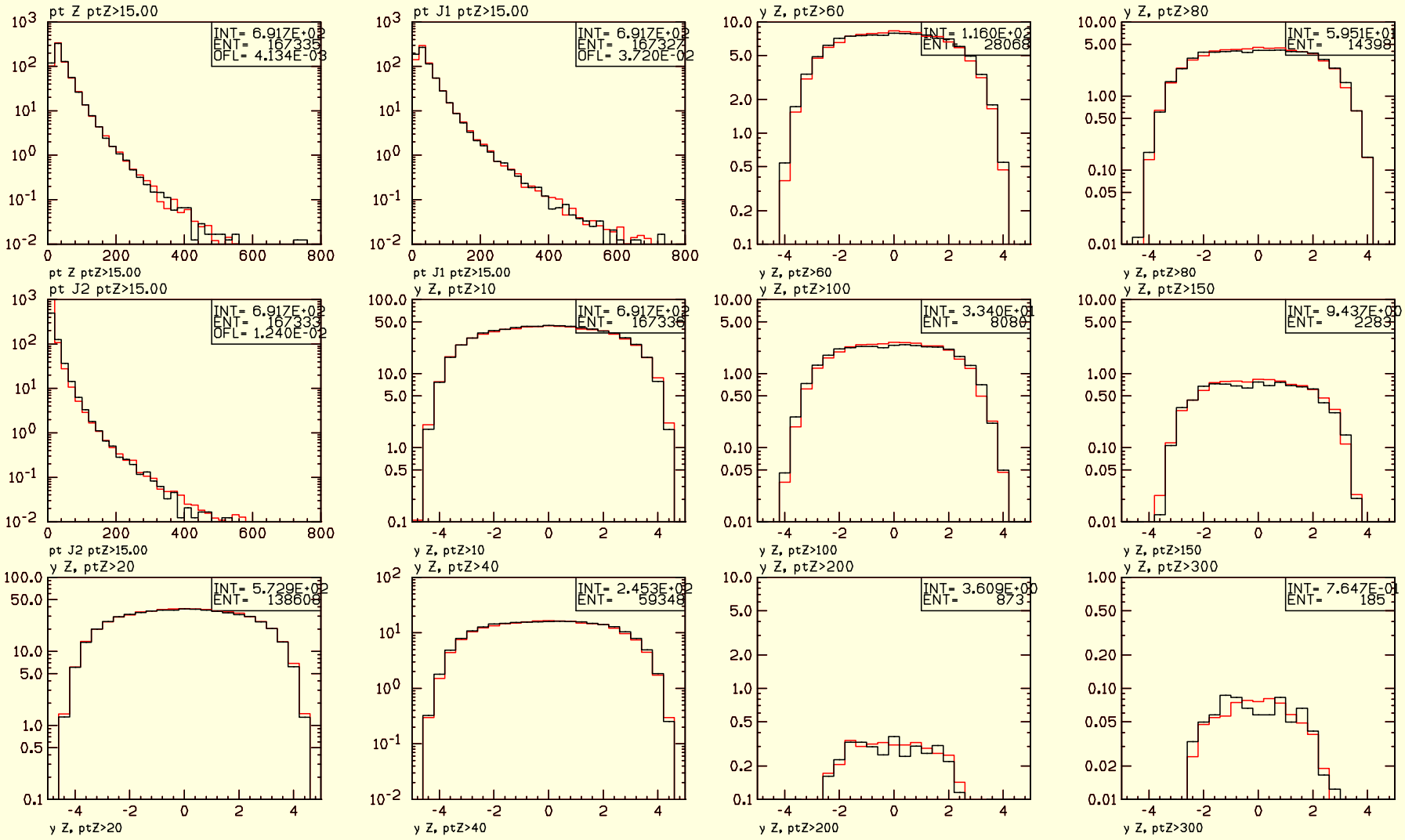

pt e- zoom
INT= 1.771E+03
ENT=489828567
OFL= 1.444E+01
pt e- zoom

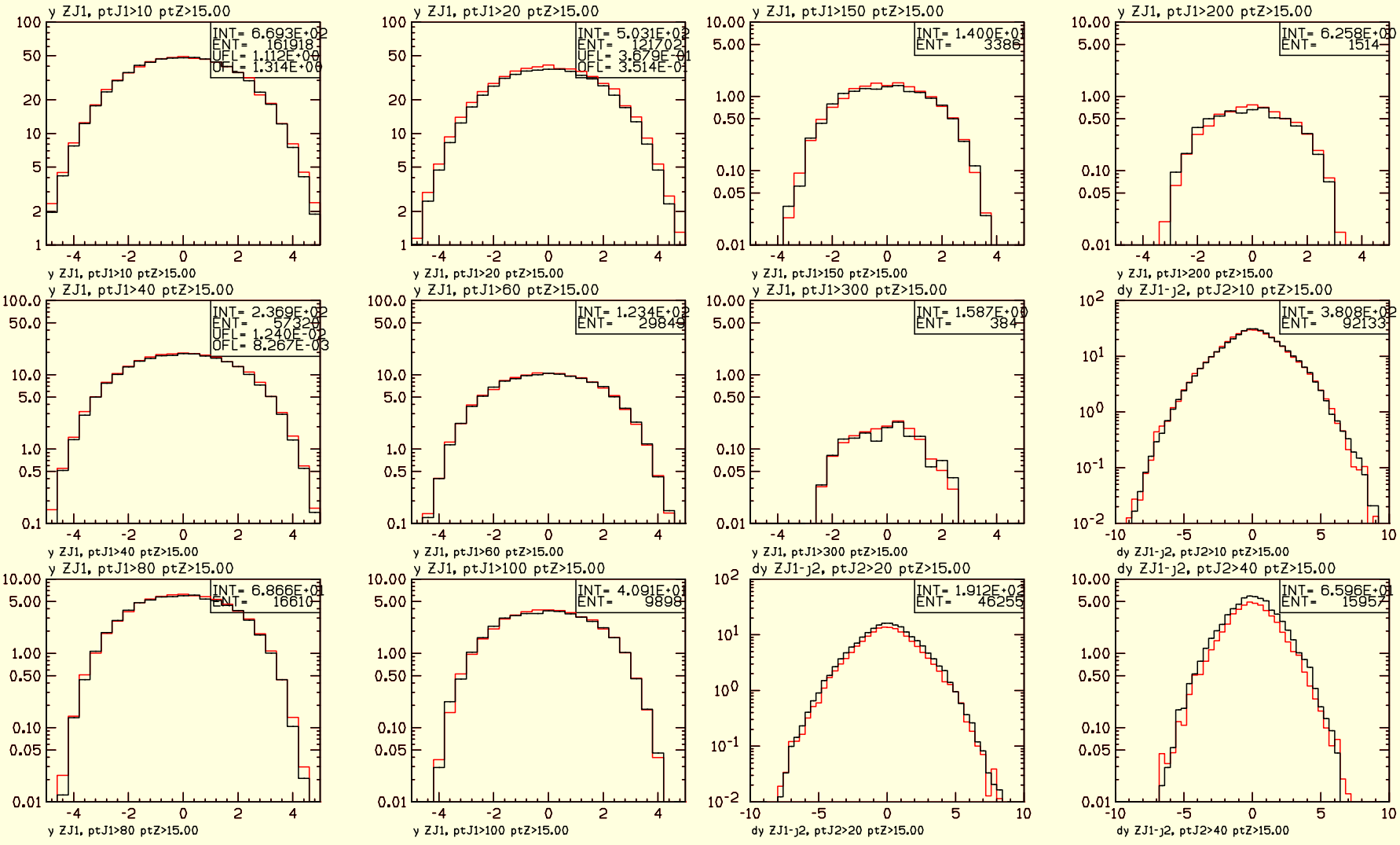

pt e+ zoom
INT= 1.772E+03
ENT=489729894
OFL= 1.379E+01
pt e+ zoom

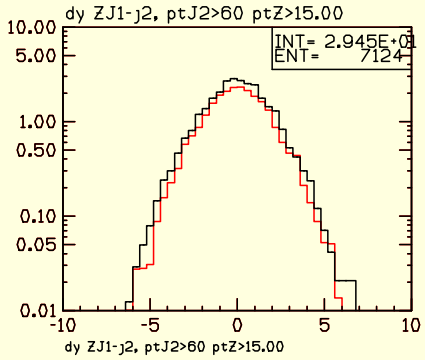


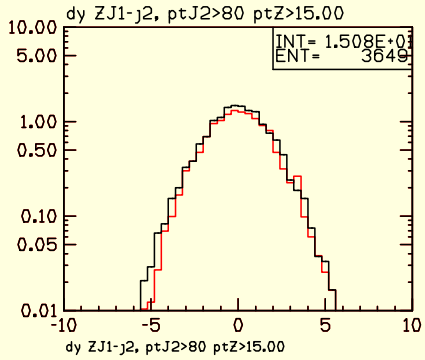
Everything seems to work ...

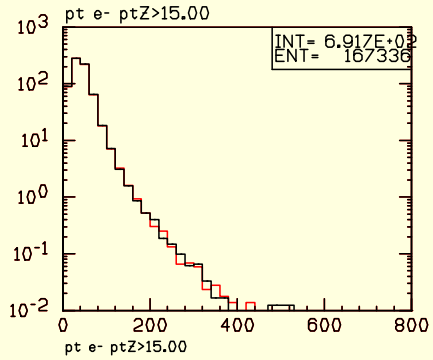
Now compare POWHEG+HERWIG with NLO (red)

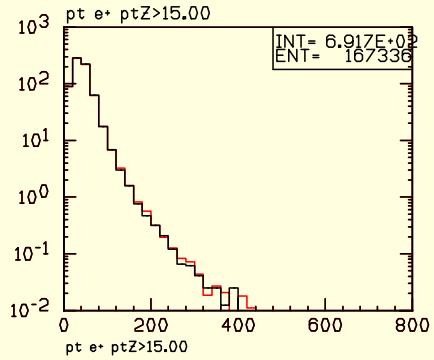


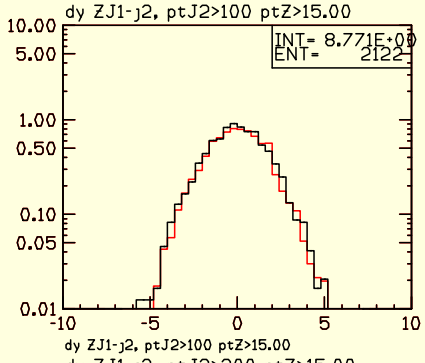

y ZJ1, ptJ1>10 ptZ>15.00
INT= 6.693E+02
ENT= 161918
UFL= 1.112E+00
OFL= 1.314E+00
y ZJ1, ptJ1>10 ptZ>15.00
y ZJ1, ptJ1>20 ptZ>15.00
INT= 5.031E+02
ENT= 121702
UFL= 3.679E-01
OFL= 3.514E-01
y ZJ1, ptJ1>20 ptZ>15.00
y ZJ1, ptJ1>150 ptZ>15.00
INT= 1.400E+01
ENT= 3386
y ZJ1, ptJ1>150 ptZ>15.00
y ZJ1, ptJ1>200 ptZ>15.00
INT= 6.258E+00
ENT= 1514
y ZJ1, ptJ1>200 ptZ>15.00
y ZJ1, ptJ1>40 ptZ>15.00
INT= 2.369E+02
ENT= 57320
UFL= 1.240E-02
OFL= 8.267E-03
y ZJ1, ptJ1>40 ptZ>15.00
y ZJ1, ptJ1>60 ptZ>15.00
INT= 1.234E+02
ENT= 29849
y ZJ1, ptJ1>60 ptZ>15.00
y ZJ1, ptJ1>300 ptZ>15.00
INT= 1.587E+00
ENT= 384
y ZJ1, ptJ1>300 ptZ>15.00
dy ZJ1-j2, ptJ2>10 ptZ>15.00
INT= 3.808E+02
ENT= 92133
dy ZJ1-j2, ptJ2>10 ptZ>15.00
y ZJ1, ptJ1>80 ptZ>15.00
INT= 6.866E+01
ENT= 16610
y ZJ1, ptJ1>80 ptZ>15.00
y ZJ1, ptJ1>100 ptZ>15.00
INT= 4.091E+01
ENT= 9898
y ZJ1, ptJ1>100 ptZ>15.00
dy ZJ1-j2, ptJ2>20 ptZ>15.00
INT= 1.912E+02
ENT= 46255
dy ZJ1-j2, ptJ2>20 ptZ>15.00
dy ZJ1-j2, ptJ2>40 ptZ>15.00
INT= 6.596E+01
ENT= 15957
dy ZJ1-j2, ptJ2>40 ptZ>15.00

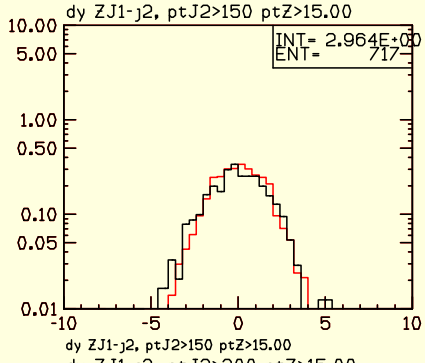

dy ZJ1-j2, ptJ2>60 ptZ>15.00
INT= 2.945E+01
ENT= 7124
dy ZJ1-j2, ptJ2>60 ptZ>15.00

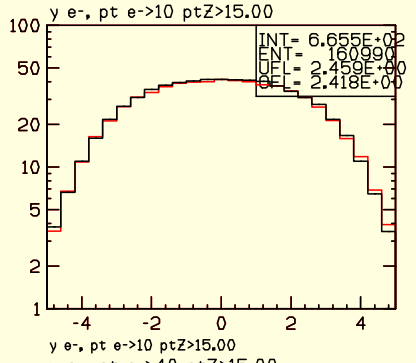

dy ZJ1-j2, ptJ2>80 ptZ>15.00
INT= 1.508E+01
ENT= 3649
dy ZJ1-j2, ptJ2>80 ptZ>15.00

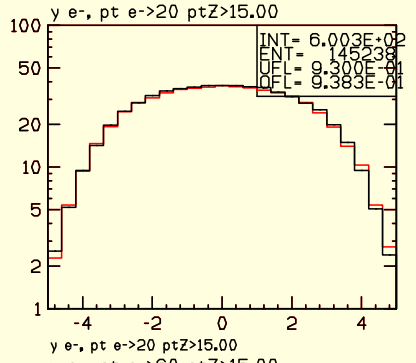

pt e- ptZ>15.00
INT= 6.917E+02
ENT= 167336
pt e- ptZ>15.00

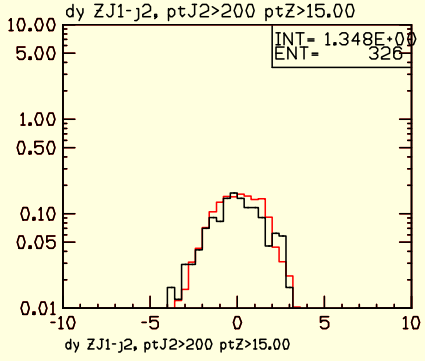

pt e+ ptZ>15.00
INT= 6.917E+02
ENT= 167336
pt e+ ptZ>15.00

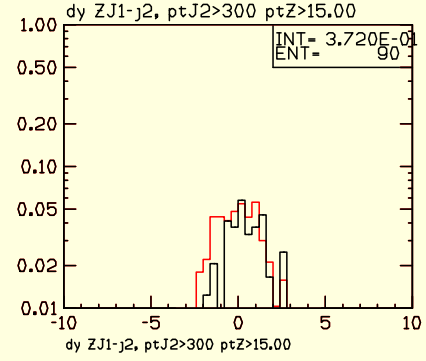

dy ZJ1-j2, ptJ2>100 ptZ>15.00
INT= 8.771E+00
ENT= 2122
dy ZJ1-j2, ptJ2>100 ptZ>15.00

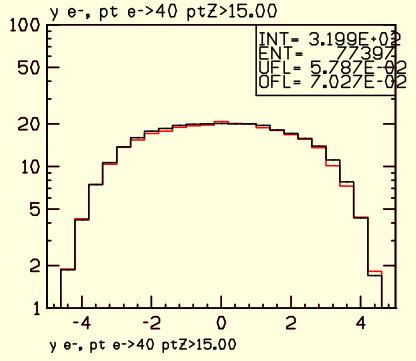

dy ZJ1-j2, ptJ2>150 ptZ>15.00
INT= 2.964E+00
ENT= 717
dy ZJ1-j2, ptJ2>150 ptZ>15.00

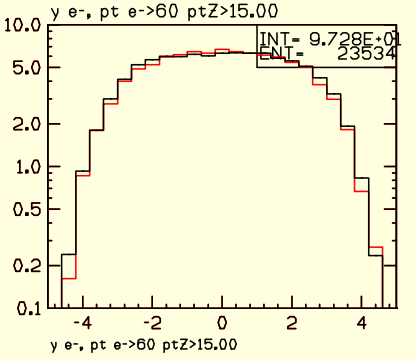

y e-, pt e->10 ptZ>15.00
INT= 6.655E+02
ENT= 160990
UFL= 2.459E+00
OFL= 2.418E+00
y e-, pt e->10 ptZ>15.00

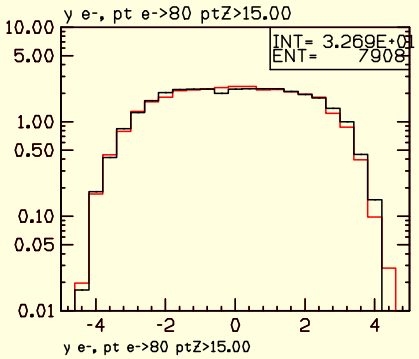

y e-, pt e->20 ptZ>15.00
INT= 6.003E+02
ENT= 145238
UFL= 9.300E-01
OFL= 9.383E-01
y e-, pt e->20 ptZ>15.00

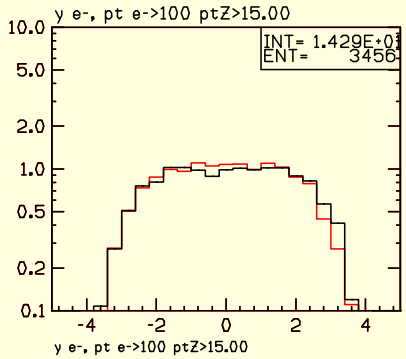

dy ZJ1-j2, ptJ2>200 ptZ>15.00
INT= 1.348E+00
ENT= 326
dy ZJ1-j2, ptJ2>200 ptZ>15.00

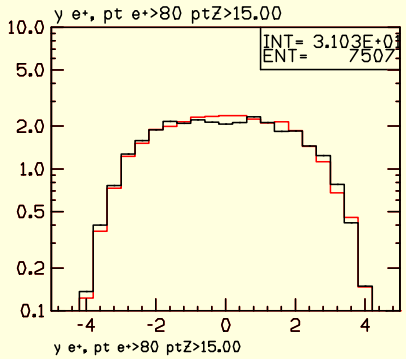

dy ZJ1-j2, ptJ2>300 ptZ>15.00
INT= 3.720E-01
ENT= 90
dy ZJ1-j2, ptJ2>300 ptZ>15.00

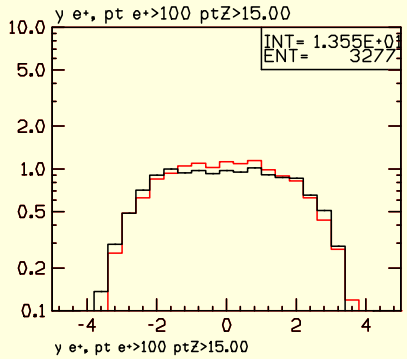

y e-, pt e->40 ptZ>15.00
INT= 3.199E+02
ENT= 77397
UFL= 5.787E-02
OFL= 7.027E-02
y e-, pt e->40 ptZ>15.00

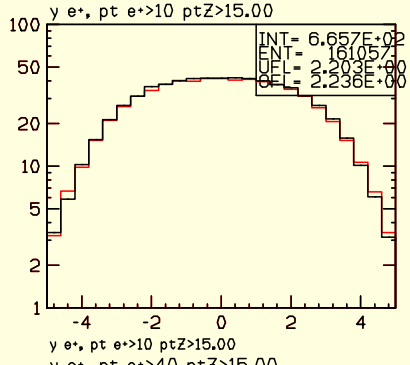

y e-, pt e->60 ptZ>15.00
INT= 9.728E+01
ENT= 23534
y e-, pt e->60 ptZ>15.00

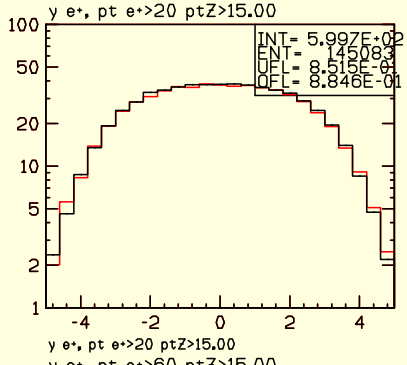

y e-, pt e->80 ptZ>15.00
INT= 3.269E+03
ENT= 7908
y e-, pt e->80 ptZ>15.00

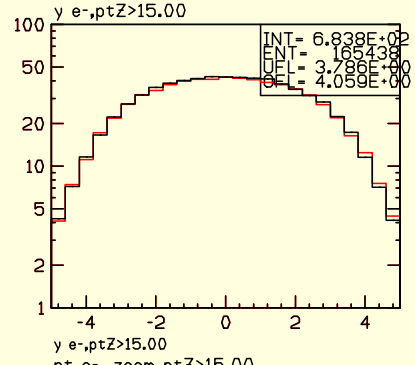

y e-, pt e->100 ptZ>15.00
INT= 1.429E+03
ENT= 3456
y e-, pt e->100 ptZ>15.00

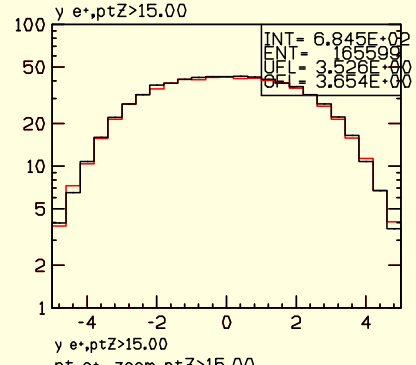

y e+, pt e+>80 ptZ>15.00
INT= 3.103E+03
ENT= 7507
y e+, pt e+>80 ptZ>15.00

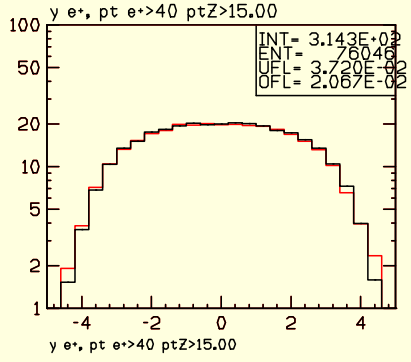

y e+, pt e+>100 ptZ>15.00
INT= 1.355E+03
ENT= 3277
y e+, pt e+>100 ptZ>15.00

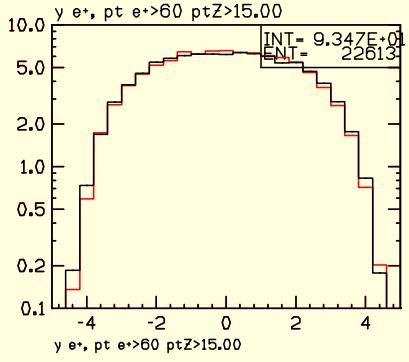

y e+, pt e+>10 ptZ>15.00
INT= 6.657E+02
ENT= 161057
UFL= 2.203E+00
OFL= 2.236E+00
y e+, pt e+>10 ptZ>15.00

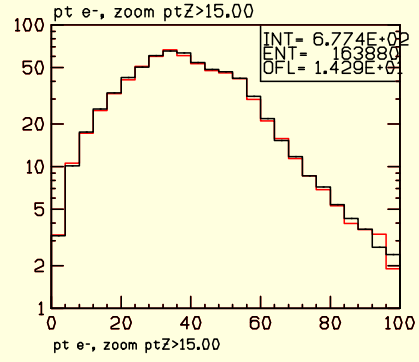

y e+, pt e+>20 ptZ>15.00
INT= 5.997E+02
ENT= 145083
UFL= 8.515E-01
OFL= 8.846E-01
y e+, pt e+>20 ptZ>15.00

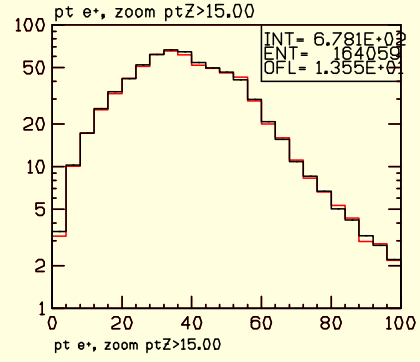

y e-,ptZ>15.00
INT= 6.838E+02
ENT= 165438
UFL= 3.786E+00
OFL= 4.059E+00
y e-,ptZ>15.00


y e+,ptZ>15.00
INT= 6.845E+02
ENT= 165599
UFL= 3.526E+00
OFL= 3.854E+00
y e+,ptZ>15.00

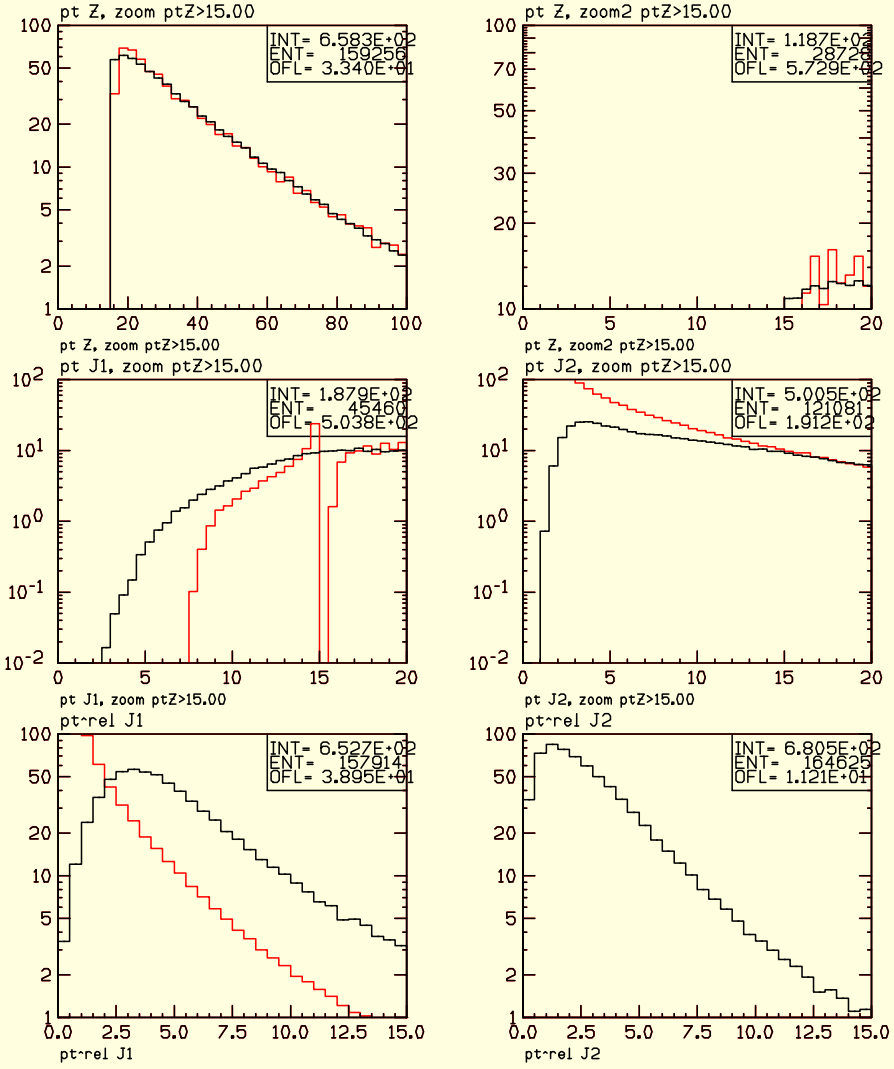

y e+, pt e+>40 ptZ>15.00
INT= 3.143E+03
ENT= 76046
UFL= 3.720E-02
OFL= 2.067E-02
y e+, pt e+>40 ptZ>15.00


y e+, pt e+>60 ptZ>15.00
INT= 9.347E+01
ENT= 22613
y e+, pt e+>60 ptZ>15.00


pt e-, zoom ptZ>15.00
INT= 6.774E+02
ENT= 163880
OFL= 1.429E+01
pt e-, zoom ptZ>15.00


pt e+, zoom ptZ>15.00
INT= 6.781E+02
ENT= 164059
OFL= 1.355E+01
pt e+, zoom ptZ>15.00

Distributions sensitive to more than two jet show noticeably different. All others in agreement with NLO



Conclusions

- New developments in the calculation and simulation of signal and background events at the LHC
- ME element programs, interfaced to Parton Shower algorithms with CKKW-like methods replace plain Parton Shower simulations.
- NLO calculations for very complex processes are becoming available
- Techniques to merge NLO calculations and showers do exist, and are being used by the experimental collaborations.